Ch 10 Inferential tools for multiple regression Chapter 11 Model Checking & Refinement

Class 17: 4/13/09 M

Slide 1 Ch 10 Inferential tools for multiple regression

Chapter 11 Model Checking & Refinement

NOTES:

HW 12 due Friday 4/17/09

Submit as Myname-HW12.doc (or *.rtf)

- HW 12 10.28: El Niño and Hurricanes
  - Due Friday 4/17/09 Noon
- Read Chapter 11: Model Checking and Refinement
  - Read chapter 11 conceptual problems & solutions
  - Post a question or response about Chapter 11 conceptual problems
- No Class Monday 4/20: Patriot's Day

Slide 2 HW 12 due Friday 4/17/09

NOTES:

Case 10.2

Energy of echolocating bats:
Do they require more energy than non-echolocating bats or birds, after accounting for the effects of body mass on energy consumption?

EEOS611

Slide 3 Case 10.2

NOTES:

Display 10.3

Mass and in-flight energy expenditure for 4 non-echolocating bats (Type = 1), 12 non-echolocating birds (Type = 2), and 4 echolocating bats (Type = 3)

Species	*Mass (g)*	*Type*	*Flight Energy Expenditure (W)*
Pteropus gouldii	779	1	43.7
Pteropus poliocephalus	628	1	34.8
Hypsignathus monstrosus	258	1	23.3
Eidolon helvum	315	1	22.4
Meliphaga virescens	24.3	2	2.46
Melipsittacus undulatus	35	2	3.93
Sturnus vulgaris	72.8	2	9.15
Falco spaverius	120	2	13.8
Falco tinnunculus	213	2	14.6
Corvus ossifragus	275	2	22.8
Larus atricilla	370	2	26.2
Columba livia	384	2	25.9
Columba livia	442	2	29.5
Columba livia	412	2	43.7
Columba livia	330	2	34.0
Corvus crytoleucos	480	2	27.8
Phyllostomas hastatus	93	3	8.83
Plecotus auritus	8	3	1.35
Pipistrellus pipistrellus	6.7	3	1.02
Plecotus auritus	7.7	3	1.02

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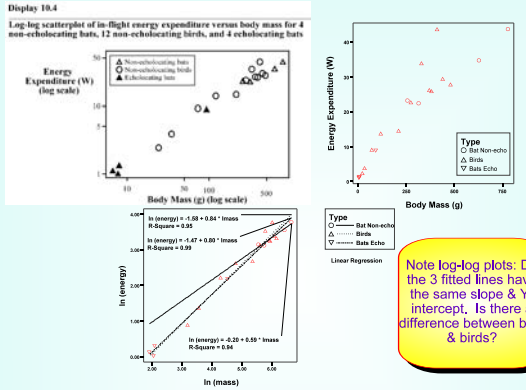
Slide 4 Display 10.3

NOTES:

Display 10.4

Log-log scatterplot of in-flight energy expenditure versus body mass for 4 non-echolocating bats, 12 non-echolocating birds, and 4 echolocating bats



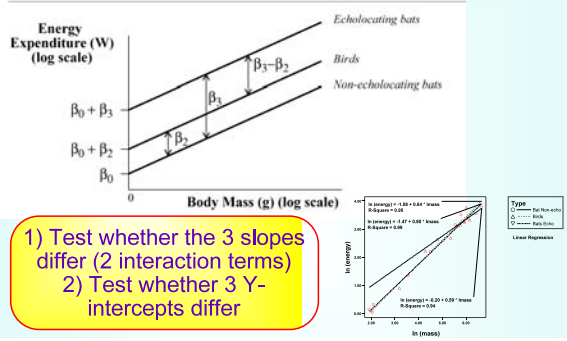
Note log-log plots: Do the 3 fitted lines have the same slope & Y-intercept. Is there a difference between bats & birds?

Slide 5 Display 10.4

NOTES:

Display 10.5

The parallel regression lines model for the bat echolocation data



1) Test whether the 3 slopes differ (2 interaction terms)
2) Test whether 3 Y-intercepts differ

Slide 6 Display 10.5

NOTES:

Display 10.12

The extra sum of squares F-test comparing the separate regression lines model to the parallel regression lines model; bat echolocation data

The models differ in 2 interaction terms

(1) *FIT FULL MODEL:* $\mu\{energy \mid lmass, TYPE\} = \beta_0 + \beta_1 lmass + \beta_2 bird + \beta_3 ebat + \beta_4\, lmass \times bird + \beta_5\, lmass \times ebats$

Source of Variation	Sum of Squares	df	Mean Square	F-Statistic	p-value
Regression	29.46993	5	5.89399	163.4	<.0001
Residual	.50487	14	.03606		
Total	29.97480	19			

Residual SS; Estimate of σ^2; d.f.

(2) *FIT REDUCED MODEL:* $\mu\{energy \mid lmass, TYPE\} = \beta_0 + \beta_1 lmass + \beta_2 bird + \beta_3 ebat$

Source of Variation	Sum of Squares	df	Mean Square	F-Statistic	p-value
Regression	29.42148	3	9.80716	283.6	<.0001
Residual	.55332	16	.03458		
Total	29.97480	19			

Residual SS

…

Slide 7 Display 10.12

NOTES:

Display 10.12

The extra sum of squares F-test comparing the separate regression lines model to the parallel regression lines model; bat echolocation data

…

(3) *The extra sum of squares is the difference between residual sums of squares*

Extra SS = .55332 - .50487 = .04845

(5) *Calculate the F-Statistic* → F-Statistic = $\left[\frac{.04845}{2}\right] / .03606 = .672$

Numerator d.f. = number of β's in full model minus number of β's in reduced model

(6) *Look up* $Pr(F_{2,14} > 0.672)$ → p-value = 0.53

Conclusion: *There is no evidence that the association between energy expenditure and body size is different for the three types of flying vertebrates (p-value = 0.53).*

Slide 8

NOTES:

t tests: Are interaction terms zero?

Parallel slopes model, Can't rely on the t statistic alone to judge whether both interaction terms can be dropped

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	-1.468	.137		-10.705	3.1E-009	-1.76	-1.18
	ln (mass)	.809	.027	.990	30.127	7.4E-017	.75	.86
2	(Constant)	-1.576	.287		-5.488	5.0E-005	-2.19	-.97
	ln (mass)	.815	.045	.998	18.297	3.8E-012	.72	.91
	Birds	.102	.114	.041	.896	.384	-.14	.34
	Echolocating bats	.079	.203	.026	.388	.703	-.35	.51
3	(Constant)	-.202	1.261		-.161	.875	-2.91	2.50
	ln (mass)	.590	.206	.722	2.861	.013	.15	1.03
	Birds	-1.378	1.295	-.552	-1.064	.305	-4.16	1.40
	Echolocating bats	-1.268	1.285	-.414	-.987	.341	-4.03	1.49
	Intxn (Mass, Birds v. Bats)	.246	.213	.536	1.151	.269	-.21	.70
	Intxn (Mass,Echolocating bats)	.215	.224	.204	.961	.353	-.26	.69

a. Dependent Variable: ln (energy)

Two 1-coefficient-at-a-time t tests can have p values > 0.05, but both together can have p<0.05. Need extra Sum of Squares F test with combined df in numerator.

Slide 9 t tests: Are interaction terms zero?

NOTES:

Extra sum of squares F test (=Partial F test) for both interaction terms

Model Summary[d]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics: R Square Change	F Change	df1	df2	Sig. F Change
1	.990[a]	.981	.979	.17995	.981	907.638	1	18	.000
2	.991[b]	.982	.978	.18596	.001	.428	2	16	.659
3	.992[c]	.983	.977	.18990	.002	.672	2	14	.527

a. Predictors: (Constant), ln (mass)

b. Predictors: (Constant), ln (mass), Birds not Bats, Bats (echo) v. Bats (non-echo)

c. Predictors: (Constant), ln (mass), Birds not Bats, Bats (echo) v. Bats (non-echo), lntxn (Mass,Twobats), lntxn (Mass, Birds v. Bats)

d. Dependent Variable: ln (Energy)

ANOVA[d]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	29.392	1	29.392	907.638	7.44E-017[a]
	Residual	.583	18	.032		
	Total	29.975	19			
2	Regression	29.421	3	9.807	283.589	4.46E-014[b]
	Residual	.553	16	.035		
	Total	29.975	19			
3	Regression	29.470	5	5.894	163.440	6.70E-012[c]
	Residual	.505	14	.036		
	Total	29.975	19			

a. Predictors: (Constant), ln (mass)

b. Predictors: (Constant), ln (mass), Birds not Bats, Bats (echo) v. Bats (non-echo)

c. Predictors: (Constant), ln (mass), Birds not Bats, Bats (echo) v. Bats (non-echo), lntxn (Mass,Twobats), lntxn (Mass, Birds v. Bats)

d. Dependent Variable: ln (Energy)

Slide 10 Extra sum of squares F test (=Partial F test) for both interaction terms

NOTES:

Do echolocating bats differ from non-echolocating bats in energy expenditure?

Non-echolocating bats are the reference category: Little evidence (t test, p=0.7) that the difference in Y intercepts ≠ 0, so conclude that there is little evidence that echolocating and non-echolocating bats differ in energy expenditure.

Coefficients[a]

Model		Unstandardized Coefficients: B	Unstandardized Coefficients: Std. Error	Standardized Coefficients: Beta	t	Sig.	95% Confidence Interval for B: Lower Bound	95% Confidence Interval for B: Upper Bound
1	(Constant)	-1.468	.137		-10.705	.000	-1.756	-1.180
	ln (mass)	.809	.027	.990	30.127	.000	.752	.865
2	(Constant)	-1.576	.287		-5.488	.000	-2.185	-.967
	ln (mass)	.815	.045	.998	18.257	.000	.721	.909
	Birds	.102	.114	.041	.896	.384	-.140	.344
	Echolocating bats	.079	.203	.026	.388	.703	-.351	.508
3	(Constant)	-.202	1.261		-.161	.875	-2.908	2.503
	ln (mass)	.590	.206	.722	2.861	.013	.148	1.032
	Birds	-1.378	1.295	-.552	-1.064	.305	-4.156	1.400
	Echolocating bats	-1.268	1.285	-.414	-.987	.341	-4.025	1.489
	Bird x mass	.246	.213	.536	1.151	.269	-.212	.703
	Ebat x mass	.215	.224	.204	.961	.353	-.265	.694

a. Dependent Variable: ln (Energy)

Slide 11 Do echolocating bats differ from non-echolocating bats in energy expenditure?

NOTES:

Display 10.10

The extra-sum-of-squares F-test for testing equality of intercepts in the parallel regression lines model; bat echolocation data

1. *Fit the FULL model: $\mu_{|energy\,|\,lmass,\,TYPE}$ = $\beta_0 + \beta_1 lmass + \beta_2 bird + \beta_3 ebat$*

   Sum of squared residuals = .55332 **d.f. = 16** $\hat{\sigma}^2 = .03458$

2. *Fit the REDUCED model: $\mu_{|energy\,|\,lmass,\,TYPE}$ = $\beta_0 + \beta_1 lmass$*

   Sum of squared residuals = .58289 **d.f. = 18**

3. *The extra-sum-of-squares is the difference between the two residual sums of squares* → **Extra SS = .58289 - .55332 = .02957**

4. *Numerator degrees of freedom are the number of β's in the full model minus the number of β's in the reduced model.* → **Numerator d.f = 4 - 2 = 2**

5. *Calculate the F-Statistic* → **F-Statistic** $= \left[\frac{\frac{.02957}{2}}{.03458}\right] = \frac{.014785}{.03458} = .428$

6. *Find $Pr(F_{2,16} > .428)$ from table, computer, or calculator* → **p-Value = .66**

Conclusion: There is no evidence that mean log energy differs for birds, echo-locating bats, and non-echolocating bats, after accounting for body mass.

Extra sum of squares F test for different Y intercepts

ANOVA[d]

Model		Sum of Squares	df	Mean Square
1	Regression	29.392	1	29.392
	Residual	.583	18	.032
	Total	29.975	19	
2	Regression	29.421	3	9.807
	Residual	.553	16	.035
	Total	29.975	19	

Slide 12 Display 10.10

NOTES:

Display 10.6

Partial summary of the least squares fit to the regression of log energy expenditure on log body mass, an indicator variable for bird, and an indicator variable for echolocating bat

Variable	Coefficient	Standard Error	t-Statistic	2-Sided p-Value
CONSTANT	-1.5764	0.2872	5.4880	<0.0001
lmass	0.8150	0.0445	18.2966	<0.0001
bird	0.1023	0.1142	0.8956	0.3837
ebat	0.0787	0.2027	0.3881	0.7030

Estimate of σ = 0.1860, df = 16

Coefficients[a]

	Unstandardized Coefficients B	Std. Error	t	Sig.	95% Confidence Interval for B Lower Bound	Upper Bound
(Constant)	-1.576	.287	-5.488	4.96E-005	-2.19	-.97
Echolocating bats	.079	.203	.388	.703	-.35	.51
Birds	.102	.114	.896	.384	-.14	.34
ln (mass)	.815	.045	18.297	3.76E-012	.72	.91

a. Dependent Variable: ln (energy)

Slide 13 Display 10.6

NOTES:

Regression σ estimated from √Residual mean square

Model Summary[d]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics: R Square Change	F Change	df1	df2	Sig. F Change
1	.990[a]	.981	.979	.17995	.981	907.638	1	18	.000
2	.991[b]	.982	.978	.18596	.001	.428	2	16	.659
3	.992[c]	.983	.977	.18990	.002	.672	2	14	.527

a. Predictors: (Constant), ln (mass)

b. Predictors: (Constant), ln (mass), Birds not Bats, Bats (echo) v. Bats (non-echo)

c. Predictors: (Constant), ln (mass), Birds not Bats, Bats (echo) v. Bats (non-echo), lntxn (Mass,Twobats), lntxn (Mass, Birds v. Bats)

d. Dependent Variable: ln (Energy)

ANOVA[d]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	29.392	1	29.392	907.638	7.44E-017[a]
	Residual	.583	18	.032		
	Total	29.975	19			
2	Regression	29.421	3	9.807	283.589	4.46E-014[b]
	Residual	.553	16	.035		
	Total	29.975	19			
3	Regression	29.470	5	5.894	163.440	6.70E-012[c]
	Residual	.505	14	.036		
	Total	29.975	19			

a. Predictors: (Constant), ln (mass)

b. Predictors: (Constant), ln (mass), Birds not Bats, Bats (echo) v. Bats (non-echo)

c. Predictors: (Constant), ln (mass), Birds not Bats, Bats (echo) v. Bats (non-echo), lntxn (Mass,Twobats), lntxn (Mass, Birds v. Bats)

d. Dependent Variable: ln (Energy)

Note that mean squares are variances, the square root of the residual mean square provides the standard error for the regression.

Slide 14 Regression σ estimated from √Residual mean square

NOTES:

Display 10.6

Partial summary of the least squares fit to the regression of log energy expenditure on log body mass, an indicator variable for bird, and an indicator variable for echolocating bat

Variable	Coefficient	Standard Error	t-Statistic	2-Sided p-Value
CONSTANT	-1.5764	0.2872	5.4880	<0.0001
lmass	0.8150	0.0445	18.2966	<0.0001
bird	0.1023	0.1142	0.8956	0.3837
ebat	0.0787	0.2027	0.3881	0.7030

Estimate of σ = 0.1860, df = 16

How do you estimate sigma, σ, standard error of the estimate, root mean square error for the regression, standard error for the regression?

Slide 15

NOTES:

Slide 16

Model Summary[d]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	R Square Change	F Change	df1	df2	Sig. F Change
1	.990[a]	.981	.979	.17995	.981	907.638	1	18	.000
2	.991[b]	.982	.978	.18596	.001	.428	2	16	.659
3	.992[c]	.983	.977	.18990	.002	.672	2	14	.527

Change Statistics

a. Predictors: (Constant), ln (mass)

b. Predictors: (Constant), ln (mass), Birds not Bats, Bats (echo) v. Bats (non-echo)

c. Predictors: (Constant), ln (mass), Birds not Bats, Bats (echo) v. Bats (non-echo), lntxn (Mass,Twobats), lntxn (Mass, Birds v. Bats)

d. Dependent Variable: ln (Energy)

ANOVA[d]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	29.392	1	29.392	907.638	7.44E-017[a]
	Residual	.583	18	.032		
	Total	29.975	19			
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b. Predictors: (Constant), ln (mass), Birds not Bats, Bats (echo) v. Bats (non-echo)

c. Predictors: (Constant), ln (mass), Birds not Bats, Bats (echo) v. Bats (non-echo), lntxn (Mass,Twobats), lntxn (Mass, Birds v. Bats)

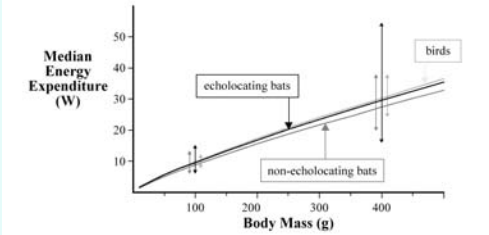
d. Dependent Variable: ln (Energy)

NOTES:

Slide 17 Display 10.8

Display 10.8

Estimated median energy expenditures for birds, echolocating bats, and non-echolocating bats as functions of body mass; parallel lines model on log-log scale, with 95% confidence bands



Working-Hotelling CI, using Scheffé multiplier

NOTES:

Slide 18 Display 10.9

Display 10.9

Construction of the 95% confidence band using repeated fits of the multiple regression model with different reference points

① *Computer Work*

Reference Point TYPE	Body Mass	Explanatory Variables TYPE indicators	Body Mass Variable	Intercept Estimate	Standard Error
birds	100	*nbat, ebat*	*lmass* - log(100)	2.2789	0.0604
	400	" "	*lmass* - log(400)	3.4087	0.0635
non-echo bats	100	*ebat, bird*	*lmass* - log(100)	2.1767	0.1144
	400	" "	*lmass* - log(400)	3.3064	0.0931
echo bats	100	*nbat, bird*	*lmass* - log(100)	2.2553	0.1277
	400	" "	*lmass* - log(400)	3.3851	0.1759

② *Hand Calculations — an Example*

$$\text{Multiplier} = \sqrt{4\,F_{4,16;\,0.95}} = 3.468$$

$$\text{Lower limit} = \exp[2.2789 - (3.468)(0.0604)] = 7.9$$

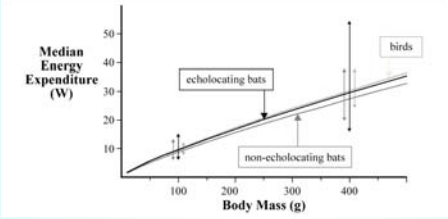
$$\text{Upper limit} = \exp[2.2789 + (3.468)(0.0604)] = 12.0$$

Note: Scheffé multiplier used for CI's: somewhat atypical, but appropriate [df=parameters in model (4), df in residual (16)]

NOTES:

Display 10.8

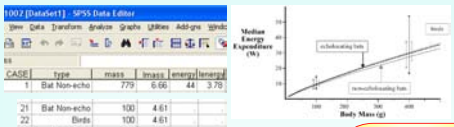
Estimated median energy expenditures for birds, echolocating bats, and non-echolocating bats as functions of body mass; parallel lines model on log-log scale, with 95% confidence bands



Scheffé multiplier produces broad confidence interval for the expected predicted value

Slide 19 Display 10.8

NOTES:



CASE	type	mass	lmass	energy	lenergy
1	Bat Non-echo	779	6.66	44	3.78
21	Bat Non-echo	100	4.61		
22	Birds	100	4.61		
23	Bats Echo	100	4.61		
24	Bat Non-echo	400	5.99		
25	Birds	400	5.99		
26	Bats Echo	400	5.99		

Th s Scheffé pred ct on nterval s based on ∞ sample s ze There s a further Scheffé adjustment for nd v dual CI's

Low95I	Up95I	Low95M	Up95M	Low95S	Up95S
30.14	73.22	38.31	57.61	33.646	65.586
5.55	14.01	6.92	11.24	5.929	13.111
6.45	14.78	8.59	11.10	7.919	12.044
5.91	15.39	7.28	12.50	6.125	14.855
17.56	42.41	22.40	33.24	19.757	37.688
19.93	45.85	26.42	34.58	24.249	37.675
17.16	50.79	20.33	42.86	16.039	54.335

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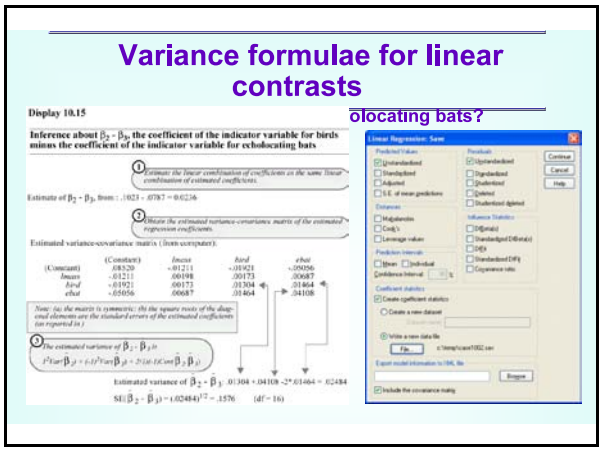
NOTES:

Syntax: Scheffé multiplier

```
* regpars is the number of parameters in the final model, with 16 df in the residual.
Compute regpars=4.
Compute residdf=16.
exe.
COMPUTE FScheffe = IDF.F(0.95,regpars,residdf) .
EXECUTE .
COMPUTE Scheffemultiplier = sqrt(regpars*FScheffe) .
EXECUTE .
* Scheffe interval is Scheffe multiplier times the standard error for each predicted value, SEP_1 was producded
by regression.
COMPUTE SchInt = SEP_1 * Scheffemultiplier .
EXECUTE .
COMPUTE L95S = PRE_1  - SchInt.
COMPUTE U95S = PRE_1 + SchInt.
EXE.
COMPUTE PredE = Exp(PRE_1).
COMPUTE Low95S  = Exp(L95S) .
COMPUTE Up95S   = Exp(U95S) .
EXECUTE .
```

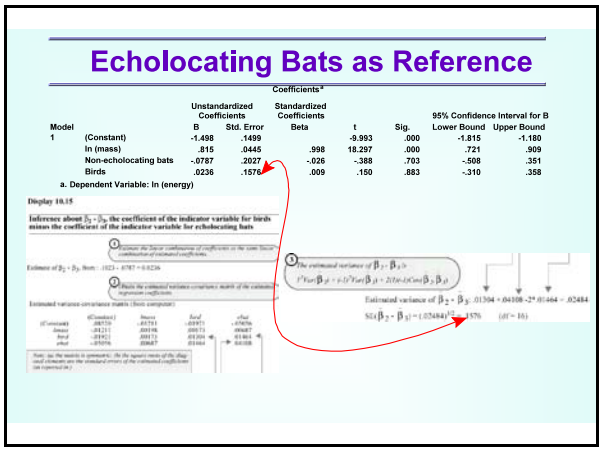
Slide 21 Syntax: Scheffé multiplier

NOTES:



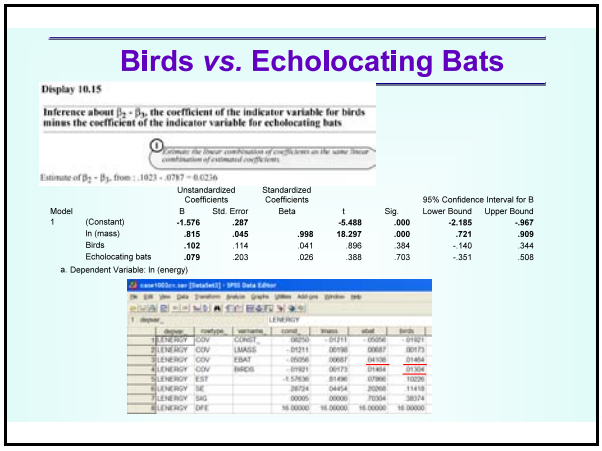
Slide 22 Variance formulae for linear contrasts

NOTES:



Slide 23 Echolocating Bats as Reference

NOTES:



Slide 24 Birds vs. Echolocating Bats

NOTES:

Using indicator variables judiciously

- By changing the reference category to echolocating bats, instead of non-echolocating bats, the bird coefficient will test the hypothesis that the bird Y-intercept differs from echolocating bat Y-intercept
  - If non-echolocating bats were the reference, the standard error for the difference in Y intercepts could be calculated using the 'propogation of error variance' formula:
  - Var(a- b)=variance(a)+variance(b)-2 cov(a,b)
- By coding the flying animal type using non-intiger coding, ebats as ½ and nebats as -½, "Birds vs. Both types of Bats" can be tested. Draper & Smith cover non-integer coding schemes.

Slide 25 Using indicator variables judiciously

NOTES:

Birds vs. Both types of bats

Weighted average of the 2 bat types using -½ and ½ for the indicator variables for ebats and bats

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	-1.537	.205		-7.481	.000	-1.973	-1.101
	ln (mass)	.815	.045	.998	18.297	.000	.721	.909
	Birds	.063	.093	.025	.676	.509	-.134	.260
	Bats (echo) v. Bats (non-echo)	.079	.203	.020	.388	.703	-.351	.508

a. Dependent Variable: ln (energy)

Statistical summary: After accounting for body mass effects, little evidence that the energy consumption by echolocating bats differs from non-echolocating bats (*t*-test, p=0.7), nor is there much evidence that birds differ from bats, echolocating or not, in energy expenditure (t test, p=0.51)

Slide 26 Birds vs. Both types of bats

NOTES:

Chapter 11 Model Checking & Refinement Case Studies

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Slide 27 Chapter 11 Model Checking & Refinement

Case Studies

NOTES:

Case Study 11.1 Gender differences in alcohol tolerance

Women have a lower alcohol tolerance & more alcohol related disease (with the same amount of drinking). Why?

- 18 Men & 14 women volunteers from Trieste
  - 3 women & 5 men were alcoholic
- Ethanol dose 0.3 g/kg orally one day & intravenously another
  - Randomly determined order
- Intravenous- oral= 1st pass metabolism
- Gastric alcohol dehydrogenase activity measured

Display 11.1

First-pass metabolism of alcohol in the stomach (mmol/liter-hour) and gastric alcohol dehydrogenase activity in the stomach (μmol/min/g of tissue) for 18 women and 14 men

Subject	Metabolism	Gastric Activity	Female (1=F, 0=M)	Alcoholic (1=A, 0=N)	Subject	Metabolism	Gastric Activity	Female (1=F, 0=M)	Alcoholic (1=A, 0=N)
1	0.6	1.0	1	1	17	2.5	3.0	1	0
2	0.6	1.0	1	1	18	2.9	2.2	1	0
3	1.5	1.3	1	1	19	1.5	1.3	0	1
4	0.4	2.2	1	0	20	1.9	1.2	0	1
5	0.1	1.1	1	0	21	2.7	1.4	0	1
6	0.2	1.2	1	0	22	3.0	1.3	0	1
7	0.3	0.9	1	0	23	3.7	2.7	0	1
8	0.3	0.8	1	0	24	0.3	1.1	0	0
9	0.4	1.3	1	0	25	2.3	2.3	0	0
10	1.0	0.9	1	0	26	2.7	2.7	0	0
11	1.1	1.6	1	0	27	3.0	1.4	0	0
12	1.2	1.7	1	0	28	4.0	2.2	0	0
13	1.3	1.7	1	0	29	4.5	2.0	0	0
14	1.6	2.2	1	0	30	6.1	2.4	0	0
15	1.8	0.8	1	0	31	9.5	5.1	0	0
16	2.9	2.0	1	0	32	12.3	4.1	0	0

Slide 28 Case Study 11.1 Gender differences in alcohol tolerance

NOTES:

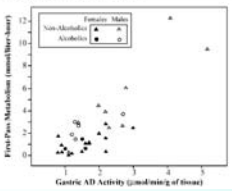
Display 11.2

Research Questions

- Do levels of first-pass metabolism differ between men and women?
- Can differences be explaine by postulating that men hav more alcohol dehydrogenas activity in their stomachs?
- Are these effects complicat by an alcoholism effect (3 women & 5 men were alcoholics)?

Display 11.2

First-pass metabolism and gastric alcohol dehydrogenase activity in alcoholic and non-alcoholic men and women

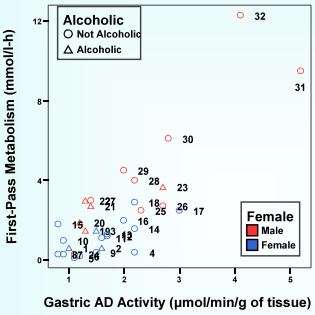


Slide 29 Display 11.2

NOTES:

SPSS Analyses

First Pass Metab. = Intravenous - oral concentration

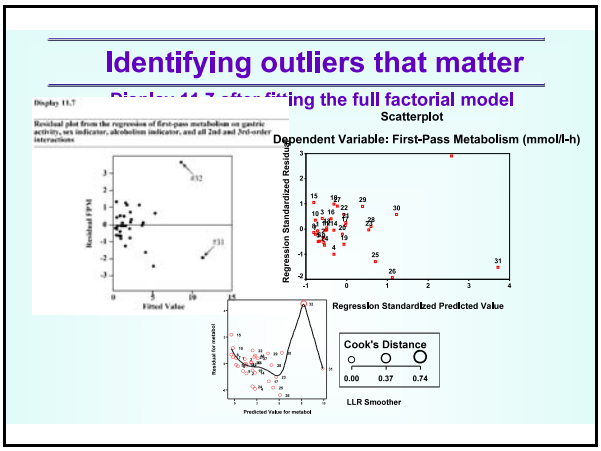


Are there outliers among these data? If so, what should be done with them?

Slide 30 SPSS Analyses

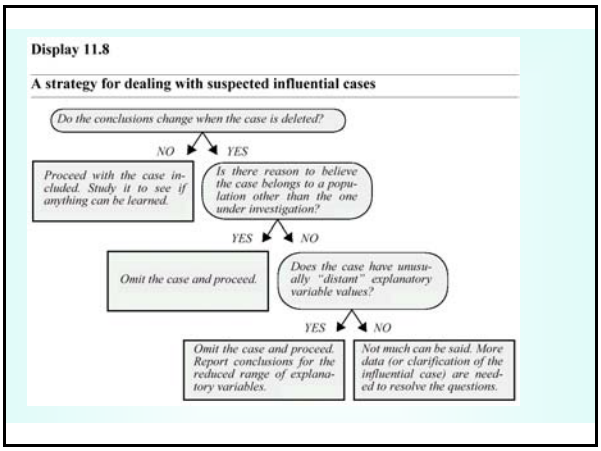
NOTES:

Slide 31 Identifying outliers that matter



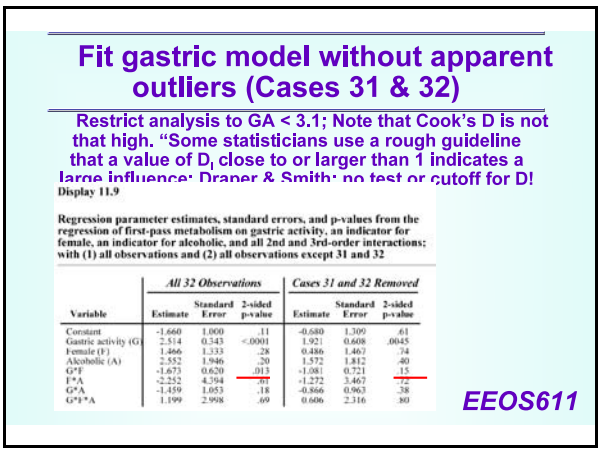
NOTES:

Slide 32



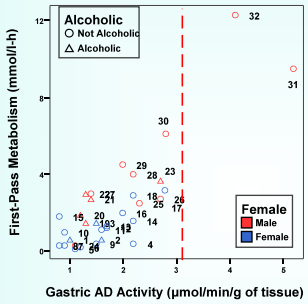
NOTES:

Slide 33 Fit gastric model without apparent outliers (Cases 31 & 32)



NOTES:

Restrict range of regression to Gastric AD levels < 3.1 µmol/min/g



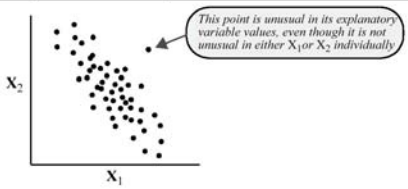
Slide 34 Restrict range of regression to Gastric AD levels < 3.1 µmol/min/g

NOTES:

Leverage

Calculated using only explanatory variables

An illustration of what is meant by "far from the average" of multiple explanatory variables when they are correlated



1/n < leverage < p/n
Potential cutoff leverage > 2p/n

Slide 35 Leverage

NOTES:

Cook's D

Detecting single observations that matter: there is no critical value for Cook's D, but D≥1 of concern; D less than 1 could be important too.

- Equivalent to performing the regression after deleting each datum one at a time
- The actual cases need not be deleted
- The change in parameters with and without the deleted case are assessed
- A point with high leverage need not affect the regression greatly

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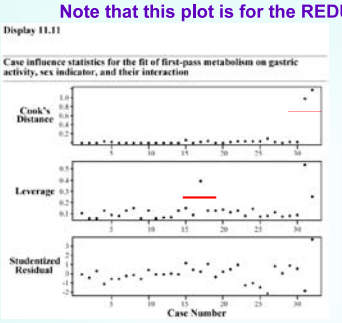
Slide 36 Cook's D

NOTES:

Display 11.11, p 318

Note that this plot is for the REDUCED model (no

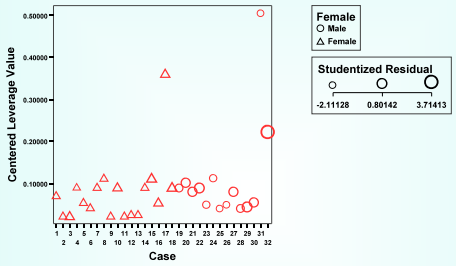


Slide 37 Display 11.11, p 318

NOTES:

SPSS analysis: Reduced model

Gastric, Gastric x Female (no β_o, no Female)



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Slide 38 SPSS analysis: Reduced model

NOTES:

Coefficients[a,b]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	-.573	.606		-.946	.352	-1.815	.668
	Gastric AD Activity (mumol/min/g of tissue)	1.449	.339	.629	4.277	.000	.755	2.143
2	(Constant)	.845	.568		1.488	.148	-.320	2.011
	Gastric AD Activity (mumol/min/g of tissue)	1.150	.271	.499	4.242	.000	.594	1.706
	Female	-1.528	.345	-.521	-4.434	.000	-2.234	-.821
3	(Constant)	.070	.802		.087	.932	-1.579	1.718
	Gastric AD Activity (mumol/min/g of tissue)	1.565	.407	.679	3.843	.001	.728	2.403
	Female	-.267	.993	-.091	-.269	.790	-2.308	1.775
	GastricxFemale	-.728	.539	-.449	-1.351	.188	-1.837	.380
4	(Constant)	-.680	1.309		-.519	.609	-3.395	2.035
	Gastric AD Activity (mumol/min/g of tissue)	1.921	.608	.833	3.159	.005	.660	3.183
	Female	.486	1.467	.166	.331	.744	-2.556	3.527
	GastricxFemale	-1.081	.721	-.666	-1.498	.148	-2.576	.415
	Alcoholic	1.572	1.812	.484	.868	.395	-2.185	5.330
	FemalexAlcohol	-1.272	3.467	-.266	-.367	.717	-8.462	5.918
	GastricxAlcohol	-.866	.963	-.428	-.899	.378	-2.863	1.132
	GastxFemxAlc	.606	2.316	.177	.262	.796	-4.197	5.408

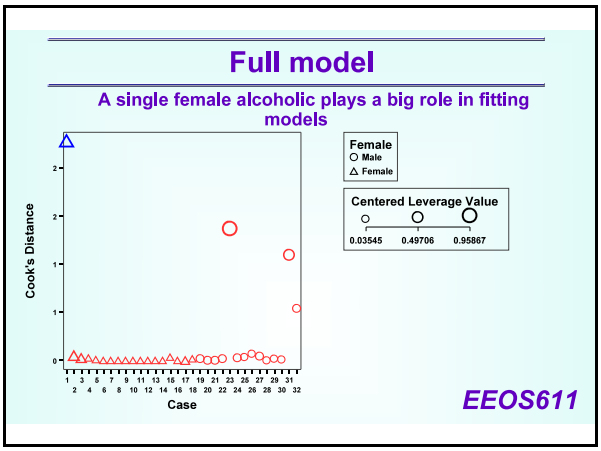
a. Dependent Variable: First-Pass Metabolism (mmol/l-h)

b. Selecting only cases for which Gastric LE 3 = 1

If there is an interaction, leave the appropriate 1st order effects in the model; Draper & Smith, Harrell (2001)

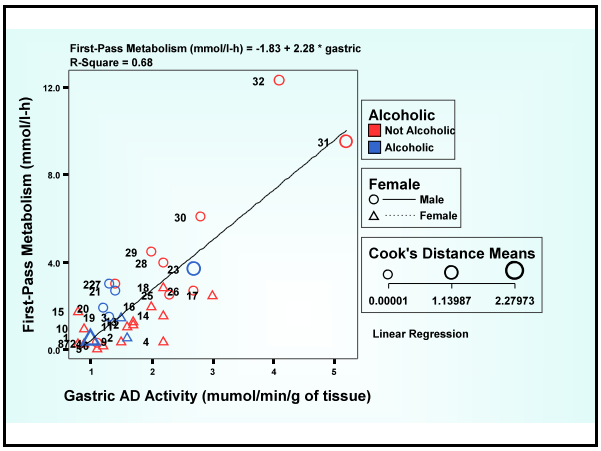
Slide 39

NOTES:



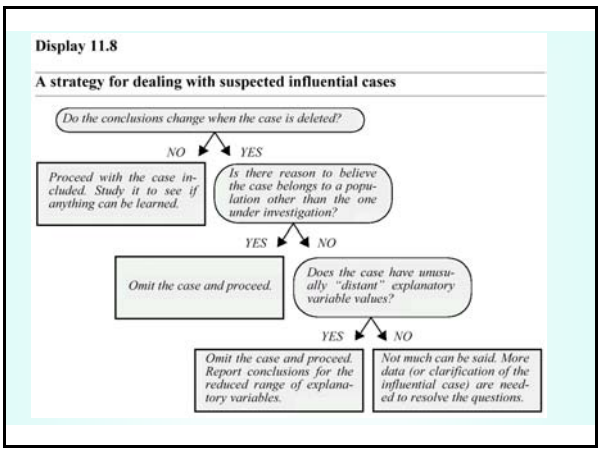
Slide 40 Full model

NOTES:



Slide 41

NOTES:



Slide 42

NOTES:

Display 11.12

Least squares estimates for the regression of first-pass metabolism on gastric AD activity, sex, and their interaction (excluding cases 31 and 32)

Variable	Estimate	Standard Error	t-Statistic	2-sided p-value
Constant	0.070	0.802	0.087	.93
gast	1.565	0.407	3.843	.0007
fem	-0.267	0.993	-0.269	.79
gast × fem	-0.728	0.539	0.114	.91

Slide 43 Display 11.12

NOTES:

Sleuth's final gastric model: Force Y intercept to be zero, drop gender main effect & delete cases with GA>3.1

Draper & Smith, Harrell (2001): it is a mistake to leave out the β_o and female main effect in this type of model! I would recommend leaving them in, even if not significant

$\mu(metabolism \mid gast, fem) = \beta_1 gast + \beta_2(gast \times fem)$.

In this model, first-pass metabolism is directly proportional to gastric activity, but the extension of proportionality differs for men and for women. The F-statistic comparing this model to the one whose fit is summarized in Display 11.13 is 0.06 with 2 and 26 degrees of freedom, so the smaller model is adequate.

The fit to the new model appears in Display 11.14. In this model, both terms are essential, so this is accepted as the final version for inference. The conclusions stated in the summary of statistical findings are based on this fit. Notice in particular that, for any level of gastric AD activity (in the range of 0.6 to 3.0), the mean first-pass metabolism for males divided by the mean first-pass metabolism for females is $\beta_1/(\beta_1 + \beta_2)$, which is estimated as 2.20. Thus, the mean for males is estimated to be 2.20 times the mean for females, even after accounting for gastric dehydrogenase activity.

11.14 Results for mean metabolism being proportional to gastric activity

Variable	Estimate	Standard error	t-statistic	Two-sided p-value
gast	1.5989	0.1240	12.800	< .0001
gast × fem	−0.8732	0.1740	−5.019	< .0001

Residual SD = 0.9518; d.f. = 26

Slide 44 Sleuth's final gastric model: Force Y intercept to be zero, drop gender main effect & delete cases with GA>3.1

NOTES:

Sleuth's final Case 11.1 model: an interaction, but without a Y intercept or Female main effect

Coefficients[a,b,c]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	Gastric AD Activity (mumol/min/g of tissue)	1.599	.125	1.218	12.800	.000	1.343	1.855
	GastricxFemale	-.873	.174	-.478	-5.019	.000	-1.230	-.517

a. Dependent Variable: First-Pass Metabolism (mmol/l-h)

b. Linear Regression through the Origin

c. Selecting only cases for which Gastric LE 3 = 1

Draper & Smith, Harrell, & I object strongly to fitting an interaction term and leaving the main effect out. It should be left in. Moreover, the intercept should usually be left in the model, even if the 95% CI includes zero

Slide 45 Sleuth's final Case 11.1 model: an interaction, but without a Y intercept or Female main effect

NOTES:

Slide 46 Gallagher's alternate model

NOTES:

Gallagher's alternate model

Gastric AD & Female main effects, then intxn ns.

Coefficients[a,b]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	-.573	.606		-.946	.352	-1.815	.668
	Gastric AD Activity (mumol/min/g of tissue)	1.449	.339	.629	4.277	.000	.755	2.143
2	(Constant)	.845	.568		1.488	.148	-.320	2.011
	Gastric AD Activity (mumol/min/g of tissue)	1.150	.271	.499	4.242	.000	.594	1.706
	Female	-1.528	.345	-.521	-4.434	.000	-2.234	-.821
3	(Constant)	.070	.802		.087	.932	-1.579	1.718
	Gastric AD Activity (mumol/min/g of tissue)	1.565	.407	.679	3.843	.001	.728	2.403
	Female	-.267	.993	-.091	-.269	.790	-2.308	1.775
	GastricxFemale	-.728	.539	-.449	-1.351	.188	-1.837	.380
4	(Constant)	-.680	1.309		-.519	.609	-3.395	2.035
	Gastric AD Activity (mumol/min/g of tissue)	1.921	.608	.833	3.159	.005	.660	3.183
	Female	.486	1.467	.166	.331	.744	-2.556	3.527
	GastricxFemale	-1.081	.721	-.666	-1.498	.148	-2.576	.415
	Alcoholic	1.572	1.812	.484	.868	.395	-2.185	5.330
	FemalexAlcohol	-1.272	3.467	-.266	-.367	.717	-8.462	5.918
	GastricxAlcohol	-.866	.963	-.428	-.899	.378	-2.863	1.132
	GastxFemxAlc	.606	2.316	.177	.262	.796	-4.197	5.408

a. Dependent Variable: First-Pass Metabolism (mmol/l-h)

b. Selecting only cases for which Gastric LE 3 = 1

Slide 47 Extra sum of squares F test

NOTES:

Extra sum of squares F test

For the alternate model, including Y intercept & Females, then gastric x females coefficient not needed

Model Summary

Model	R Gastric LE 3 = 1 (Selected)	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics R Square Change	F Change	df1	df2	Sig. F Change
1	.629[a]	.395	.374	1.1557	.395	18.292	1	28	.000
2	.806[b]	.650	.624	.8953	.255	19.659	1	27	.000
3	.820[c]	.673	.635	.8819	.023	1.824	1	26	.188
4	.828[d]	.685	.585	.9411	.012	.209	4	22	.931

a. Predictors: (Constant), Gastric AD Activity (mumol/min/g of tissue)

b. Predictors: (Constant), Gastric AD Activity (mumol/min/g of tissue), Female

c. Predictors: (Constant), Gastric AD Activity (mumol/min/g of tissue), Female, GastricxFemale

d. Predictors: (Constant), Gastric AD Activity (mumol/min/g of tissue), Female, GastricxFemale, GastxFemxAlc, GastricxAlcohol, Alcoholic, FemalexAlcohol

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Slide 48 Display 11.14 vs. Simpler model

NOTES:

Display 11.14 vs. Simpler model

Results for mean metabolism being proportional to gastric activity

Variable	Estimate	Standard Error	t-Statistic	2-sided p-value
gast	1.599	0.125	12.800	<.0001
gast × fem	-0.873	0.174	-5.019	<.0001

Coefficients[a,b]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	.070	.802		.087	.932	-1.579	1.718
	Gastric AD Activity (mumol/min/g of tissue)	1.565	.407	.679	3.843	.001	.728	2.403
	Female	-.267	.993	-.091	-.269	.790	-2.308	1.775
	GastricxFemale	-.728	.539	-.449	-1.351	.188	-1.837	.380
2	(Constant)	-.680	1.309		-.519	.609	-3.395	2.035
	Gastric AD Activity (mumol/min/g of tissue)	1.921	.608	.833	3.159	.005	.660	3.183
	Female	.486	1.467	.166	.331	.744	-2.556	3.527
	GastricxFemale	-1.081	.721	-.666	-1.498	.148	-2.576	.415
	Alcoholic	1.572	1.812	.484	.868	.395	-2.185	5.330
	FemalexAlcohol	-1.272	3.467	-.266	-.367	.717	-8.462	5.918
	GastricxAlcohol	-.866	.963	-.428	-.899	.378	-2.863	1.132
	GastxFemxAlc	.606	2.316	.177	.262	.796	-4.197	5.408

a. Dependent Variable: First-Pass Metabolism (mmol/l-h)

b. Selecting only cases for which Gastric LE 3 = 1

Leave in model

No need for interaction terms

Case 11.2

Brain barrier disruption

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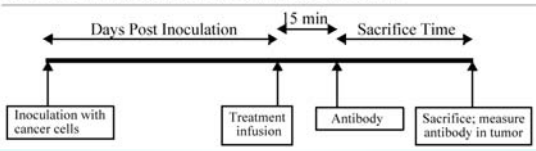
Slide 49 Case 11.2

NOTES:

Case Study 11.2 brain disruption with concentrated sugars

Display 11.3 p. 294

Time line for blood-brain barrier disruption experiment



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Slide 50 Case Study 11.2 brain disruption with concentrated sugars

NOTES:

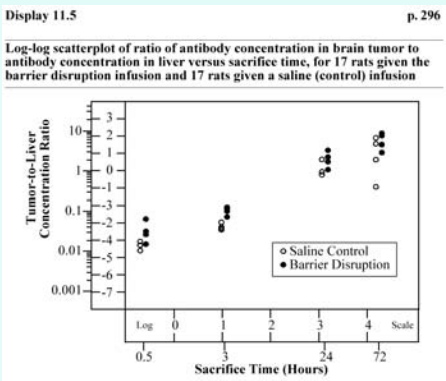
Display 11.4 p. 295

Response variable, design variables, and several covariates for 34 rats in the blood-brain barrier disruption experiment

	Response Variable	*Design Variables*		*Covariates*				
	Brain tumor Count (per gm) / Liver Count (per gm)	Sacrifice Time (hours)	Treatment	Sex	Days Post Inoculation	Tumor Weight (10^{-6} grams)	Weight Loss (grams)	Initial Weight (grams)
Case								
1	41081 / 1456164	0.5	BD	10	F	239	5.9	221
2	44286 / 1062171	0.5	BD	10	F	225	4.9	246
3	102926 / 1601936	0.5	BD	10	F	224	-4.9	61
4	28927 / 1376411	0.5	BD	10	F	184	9.8	168
5	42643 / 1351184	0.5	BD	10	F	250	6.9	164
6	31342 / 1790863	0.5	NS	10	F	196	7.7	260
7	22815 / 1633386	0.5	NS	10	F	260	0.5	27
8	16629 / 1618757	0.5	NS	10	F	273	4.0	308
9	22315 / 1567602	0.5	NS	10	F	236	2.8	93
10	77961 / 1060057	3	BD	10	F	267	2.6	73
11	73178 / 715581	3	BD	10	F	263	1.1	25
12	76167 / 620145	3	BD	10	F	228	0.0	133
13	123730 / 1068423	3	BD	9	F	261	3.4	203
14	25869 / 721436	3	NS	9	F	253	5.9	159
15	53803 / 1019352	3	NS	10	F	234	0.1	264
16	24512 / 667785	3	NS	10	F	238	0.8	34
17	50543 / 961097	3	NS	9	F	230	7.0	146
18	50690 / 1220677	3	NS	10	F	207	1.5	232
19	84616 / 48815	24	BD	10	F	254	3.9	155
20	55153 / 16885	24	BD	10	M	256	-4.7	190
21	48829 / 22395	24	BD	10	M	247	-2.8	161
22	89454 / 83504	24	BD	11	F	198	4.2	214
23	37928 / 20325	24	NS	10	F	237	2.5	224
24	12816 / 15985	24	NS	10	M	293	3.1	151
25	23734 / 25895	24	NS	10	M	288	9.7	285
26	31097 / 33224	24	NS	11	F	236	5.9	180
27	35395 / 4142	72	BD	11	F	251	4.1	39
28	18270 / 2364	72	BD	10	F	223	4.0	153
29	5625 / 1979	72	BD	10	M	298	12.8	164
30	7497 / 1659	72	BD	10	M	260	7.3	364
31	6250 / 928	72	NS	10	M	272	11.0	484
32	11519 / 2423	72	NS	11	F	226	2.2	168
33	3184 / 1608	72	NS	10	M	249	-4.4	191
34	1334 / 3242	72	NS	10	F	240	6.7	159

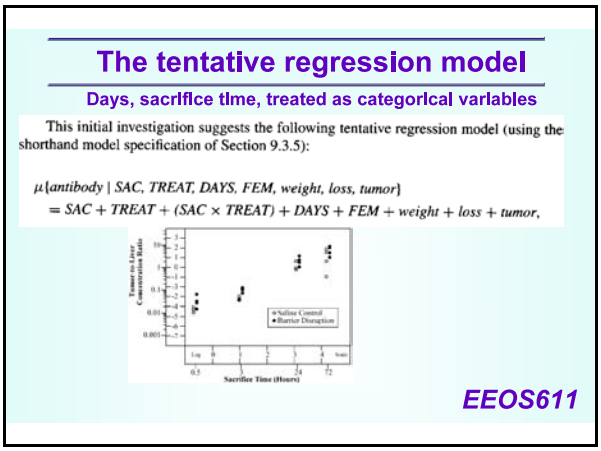
Slide 51 Display 11.4

NOTES:



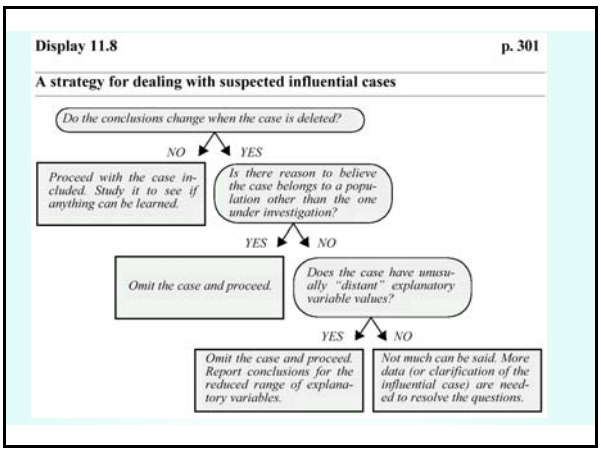
Slide 52 Display 11.5

NOTES:



Slide 53 The tentative regression model

NOTES:



Slide 54

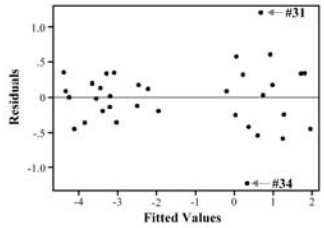
NOTES:

Brain Barrier data

Studentized residuals: Not influential, so the two data (31 & 34) left in the analysis

Display 11.6

Scatterplot of residuals versus fitted values from the fit of the logged response on a rich model for explanatory variables; brain barrier data



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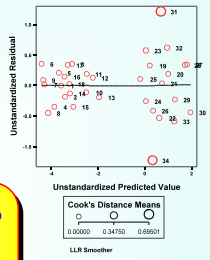
Slide 55 Brain Barrier data

NOTES:

Case 11.02

Performing the analysis described on Sleuth p 311.

```
REGRESSION
  /MISSING LISTWISE
  /STATISTICS COEFF CHA CI ANOVA
  /CRITERIA=PIN(.05) POUT(.10)
  /NOORIGIN
  /DEPENDENT lntum21v
  /METHOD=ENTER fem bd weight loss tumor
day10 day11
  /METHOD=ENTER fem bd weight loss tumor
day10 day11 sac3h sac24h sac72h
  /METHOD=ENTER fem bd weight loss tumor
day10 day11 sac3h sac24h sac72h bdx3 bdx24
bdx72
  /SCATTERPLOT=(*ZRESID ,*ZPRED )
  /SAVE PRED ZPRED COOK RESID ZRESID .
```



None of the cases seems to have sufficiently high leverage or Cook's D to be of concern;
Some indication of 'trumpet' shaped residual plot

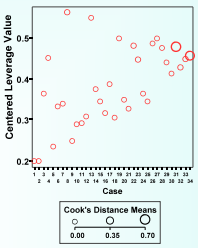
Slide 56 Case 11.02

NOTES:

Case 11.02

No need to delete cases for Case Study 11.02

- **Sleuth p. 317:**
  - p/n is the average leverage, where p is the number of parameters in the model
  - 2*p/n is sometimes used as a lower cutoff for leverage
- **For Full model for Case 11.2**
  - 13 parameter full model
  - 2*13/34=0.76
  - Only 2 data have leverage>0.6 and Max Cook's D is ≈ 0.7



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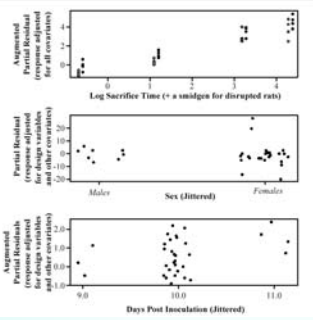
Slide 57 Case 11.02

NOTES:

Display 11.15 p. 313

Some partial residual plots for the blood-brain barrier data, with log of antibody concentration ratio (brain tumor-to-liver) as response



The effect of log sacrifice time is not linear, and the addition of a quadratic term didn't correct the lack of fit. So, sacrifice time was coded as a categorical variable, not a continuous variable.

Slide 58 Display 11.15

NOTES:

Display 11.16 p. 314

Results from the regression of log ratio of antibody concentration (brain tumor-to-liver) on sacrifice time (treated as a factor) and treatment

Variable	Estimate	Standard Error	t-Statistic	2-sided p-value
Constant	-3.505	0.195	-17.94	<.0001
Indicator for time=3	1.341	0.252	4.50	.0001
Indicator for time=24	4.257	0.259	16.43	<.0001
Indicator for time=72	5.154	0.259	19.89	<.0001
Indicator for treatment=BD	0.797	0.183	4.35	.0002

Sacrifice time was coded for as a categorical variable, using 3 (0,1) indicator or 'dummy' variables. Note that changing 'time' to a categorical variable is often an appropriate solution for temporal 'lack of fit,' as shown previously with the Boston Harbor benthic diversity analyses

Slide 59 Display 11.16

NOTES:

Case 11.02 Final model

$e^{([0.422\ 0.797\ 1.172])}$=[1.53 2.22 3.23]

The median concentration of antibody was 2.2 times higher with sugar treatment (95% CI: 1.5, 3.2)

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	-4.302	.205		-21.010	4.299E-019	-4.720	-3.883
	Brain Disruption	.797	.183	.180	4.346	1.554E-004	.422	1.172
	3 hour sacrifice	1.134	.252	.226	4.501	1.012E-004	.619	1.650
	24 hour sacrifice	4.257	.259	.815	16.431	3.131E-016	3.727	4.787
	72 hour sacrifice	5.154	.259	.987	19.892	1.902E-018	4.624	5.684
2	(Constant)	-4.930	3.096		-1.593	.124	-11.320	1.459
	Brain Disruption	.831	.198	.188	4.206	3.125E-004	.423	1.239
	3 hour sacrifice	1.089	.294	.217	3.705	.001	.483	1.696
	24 hour sacrifice	4.114	.337	.788	12.198	5.895E-012	3.418	4.810
	72 hour sacrifice	5.137	.341	.984	15.065	9.883E-014	4.433	5.840
	Days	.019	.282	.004	.069	.946	-.553	.601
	Female	.036	.358	.007	.100	.921	-.703	.774
	Weight	.002	.005	.018	.317	.754	-.008	.011
	Loss	-.048	.028	-.091	-1.744	.094	-.105	.009
	Tumor	.001	.001	.063	1.189	.246	-.001	.004

a. Dependent Variable: Ln(Tumor to liver)

Model Summary[c]

Model	R Square Change	F Change	df1	df2	Sig. F Change
1	.951[a]	139.646	4	29	1.671E-018
2	.006[b]	.701	5	24	.628

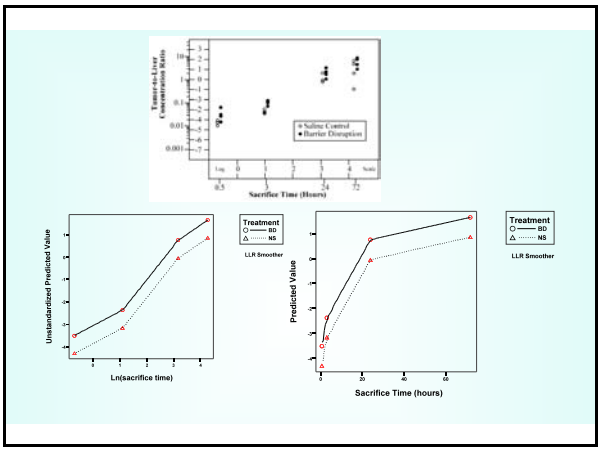
a. Predictors: (Constant), 72 hour sacrifice, Brain Disruption, 24 hour sacrifice, 3 hour sacrifice

b. Predictors: (Constant), 72 hour sacrifice, Brain Disruption, 24 hour sacrifice, 3 hour sacrifice, Loss, Weight, Tumor, Days, Female

c. Dependent Variable: Ln(Tumor to liver)

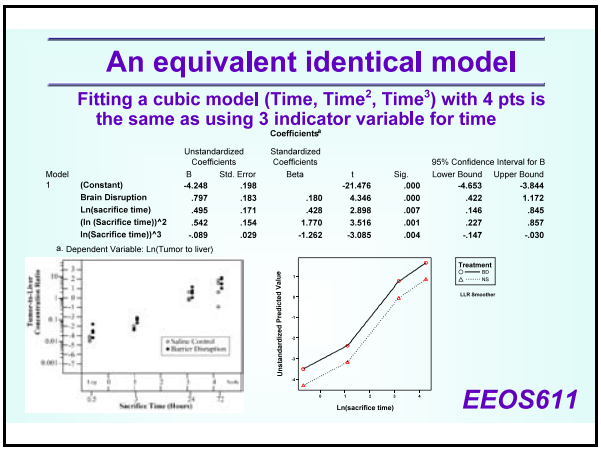
Slide 60 Case 11.02 Final model

NOTES:



Slide 61

NOTES:



An equivalent identical model

Fitting a cubic model (Time, Time^2, Time^3) with 4 pts is the same as using 3 indicator variable for time

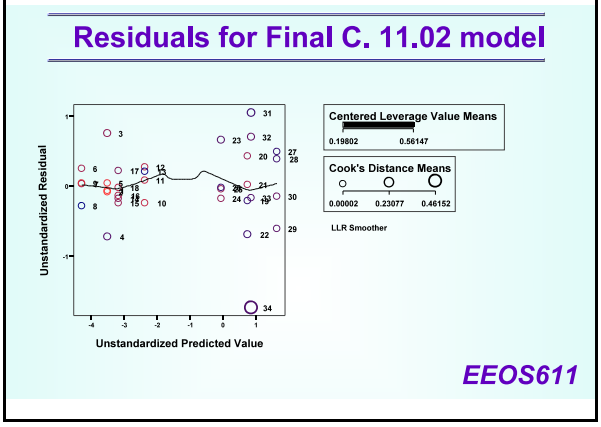
Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	-4.248	.198		-21.476	.000	-4.653	-3.844
	Brain Disruption	.797	.183	.180	4.346	.000	.422	1.172
	Ln(sacrifice time)	.495	.171	.428	2.898	.007	.146	.845
	(ln (Sacrifice time))^2	.542	.154	1.770	3.516	.001	.227	.857
	ln(Sacrifice time))^3	-.089	.029	-1.262	-3.085	.004	-.147	-.030

a. Dependent Variable: Ln(Tumor to liver)

Slide 62 An equivalent identical model

NOTES:



Residuals for Final C. 11.02 model

Slide 63 Residuals for Final C. 11.02 model

NOTES:

SDFBeta

Change in each parameter estimate caused by the deletion of each data point (in standard errors)

Standardized DfBeta

$$SDBETA_{ij} = \frac{b_j - b_j(i)}{s_{(i)}\sqrt{\left(\mathbf{X}^t\mathbf{W}\mathbf{X}\right)^{-1}_{jj}}}$$

where $b_j - b_j(i)$ is the jth component of $\mathbf{b} - \mathbf{b}(i)$.

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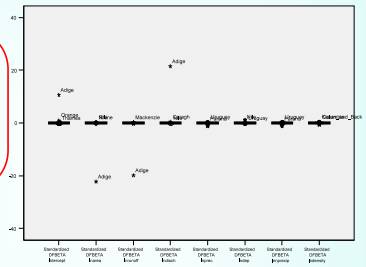
Slide 64 SDFBeta

NOTES:

SDFBeta plot, using explore

\Data\analyze\explore\ ... These are SDBETA boxplots Using 11.21 River example

Also, DFFITS, the change in predicted values with the deletion of each datum



Slide 65 SDFBeta plot, using explore

NOTES:

11.6.1 Related issues: Weighted & Nonlinear regression

- Weighted Least squares regression available in SPSS
  - /Analyze/regression/weight estimation, or WLS will provide an estimate of the weight function\
- Nonlinear regression is also available in SPSS
- Use non-linear regression, available as an advanced regression option in SPSS

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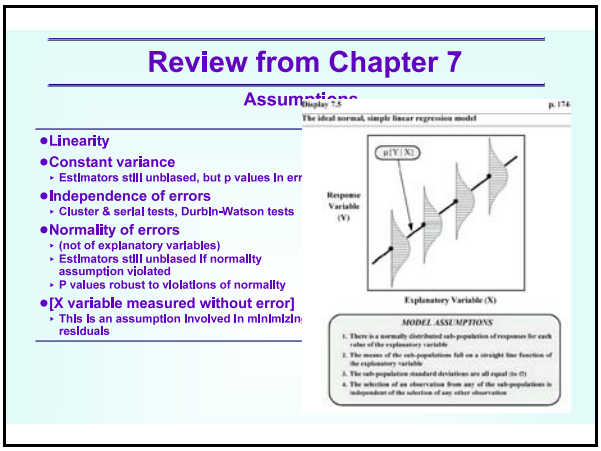
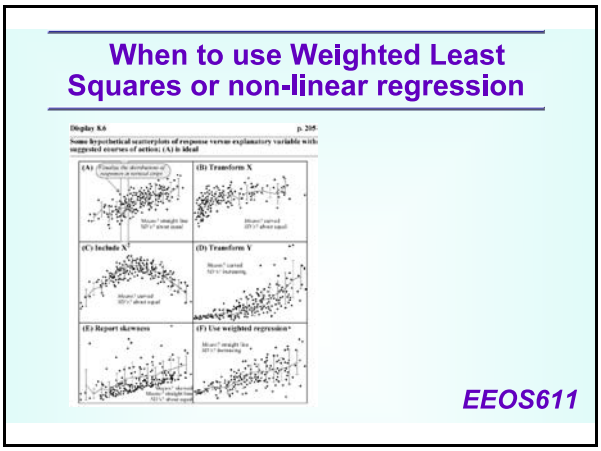
Slide 66 11.6.1 Related issues: Weighted & Nonlinear regression

NOTES:



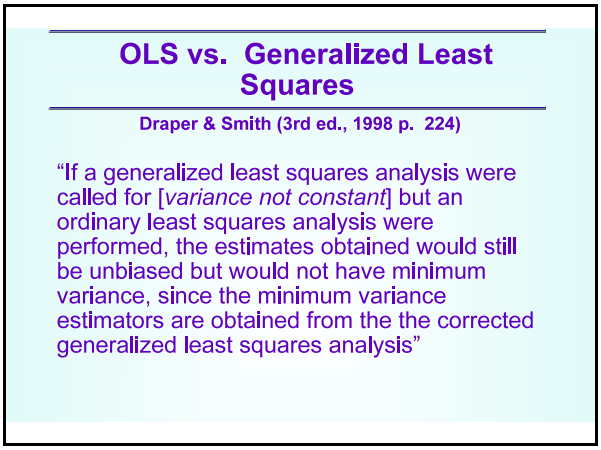
Slide 67 Review from Chapter 7

NOTES:



Slide 68 When to use Weighted Least Squares or non-linear regression

NOTES:



Slide 69 OLS vs. Generalized Least Squares

NOTES:

The Normal Equations & Matlab

$$Y = X\theta + \epsilon,$$
$$E(\epsilon) = 0.$$
$$V(\epsilon) = E(\epsilon\epsilon') = \sigma^2 I.$$

S=scalar sum of squares.

LS method requires minimization of scalar sum of squares, S.

$$S = (y - X\theta)'(y - X\theta)$$

To minimize S, set $\frac{\partial S}{\partial \theta} = 0.$

Differentiating, $2X'(y - X\theta) = 0.$

$$\hat{\theta} = (X'X)^{-1}X'y.$$

Slide 70 The Normal Equations & Matlab

NOTES:

Data with high variance are downweighted, with w_i being the scaling

Weight estimation(WLS) will find the appropriate w

Let us suppose that

$$\mathbf{V}\sigma^2 = \mathbf{V}(\mathbf{Y}) = \begin{bmatrix} 1/w_1 & & & \\ & 1/w_2 & & 0 \\ & & \ddots & \\ 0 & & & 1/w_n \end{bmatrix} \sigma^2,$$

where the w's are weights to be specified. This means that

$$\mathbf{V}^{-1} = \begin{bmatrix} w_1 & & & \\ & w_2 & & 0 \\ & & \ddots & \\ 0 & & & w_n \end{bmatrix}.$$

Applying the general results above we find, after reduction,

$$b = \frac{\sum w_i X_i Y_i}{\sum w_i X_i^2},$$

where all summations are from $i = 1, 2, \ldots, n$.

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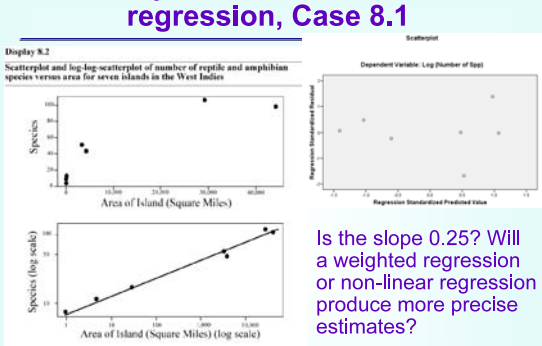
Slide 71 Data with high variance are downweighted, with wi being the scaling

NOTES:

Example of WLS & Non-linear regression, Case 8.1



Is the slope 0.25? Will a weighted regression or non-linear regression produce more precise estimates?

Slide 72 Example of WLS & Non-linear regression, Case 8.1

NOTES:

not have minimum variance, since the minimum variance estimates are obtained from the correct generalized least squares analysis.

If standard least squares is used, then the estimates are obtained from $\mathbf{b}_0 = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$ and

$$E(\mathbf{b}_0) = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{X}\boldsymbol{\beta} = \boldsymbol{\beta}$$

but

$$\mathbf{V}(\mathbf{b}_0) = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'[\mathbf{V}(\mathbf{Y})]\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}$$
$$= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\sigma^2.$$

We recall from Eq. (9.2.13) that if the correct analysis is performed,

$$\mathbf{V}(\mathbf{b}) = (\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\sigma^2$$

and, in general, elements of this matrix would provide smaller variances both for individual coefficients and for linear functions of the coefficients.

Weighted least squares regression provides higher precision for estimating parameters (and greater power for testing models)

Slide 73 Draper & Smith

NOTES: