Chapter 13: ANOVA for 2-way classifications
Start on Chapter 14: Unreplicated Factorial & Nested Designs

Class 22, 4/29/09 W

Slide 1 Chapter 13: ANOVA for 2-way classifications

Start on Chapter 14:

Unreplicated Factorial & Nested Designs

NOTES:

HW 13 due Weds 4/29/09 Noon

Submit as Myname-HW12.doc (or *.rtf)

- Read Chapter 14 Multifactor studies without replication & 16 Repeated Measures
  - We'll cover Chapter 15 (serial correlation) if there is time
- HW 14: Due Friday 5/1/09 Noon
  - 13.19 Nature Nurture
- HW15: Due Weds 5/6/09 10 am
  - 14.17 Tennessee Corn Yields
- Wimba Sessions
  - Weds night (tonight) 10 pm
  - Thursday Noon

Slide 2 HW 13 due Weds 4/29/09 Noon

NOTES:

Conclusions from last class

- Regression to the mean, the regression artefact, will be present whenever an explanatory variable (covariate) exhibits less than perfect correlation with the response variable. The higher the variability in the covariate, the more the regression to the mean effect. Including an extra explanatory variable does NOT control for the effect of that covariate
- For pre-test vs. Post-test analyses, regressing with pretest score as an explanatory variable DOES NOT remove the effects of pre-test differences.
  - Better approaches: Repeated measures designs, hierarchical linear longitudinal models, or subtract pretest from posttest (called change score analysis)

Slide 3 Conclusions from last class

NOTES:

Chapter 13

The analysis of variance for two-way classifications

Slide 4 Chapter 13

NOTES:

Goals of today's class

- Analyzing factorial models using GLM/Univariate and Regression
- What to do about interactions?
  - Note that transforms can eliminate the interaction effect
  - Sleuth doesn't properly cover the problem of pooling interaction terms (another example of Hurlbert's pseudoreplication)
    - There are rules, not covered in Sleuth, on whether the interaction SS can be pooled with the error SS
- Random vs. Fixed factors in ANOVA designs

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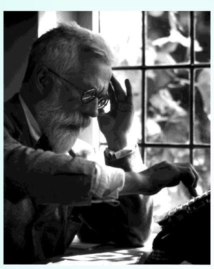
Slide 5 Goals of today's class

NOTES:

ANOVA & Factorial Designs

Ronald Fisher



"No aphorism is more frequently repeated in connection with field trials, than that we must ask Nature few questions or, ideally one question, at a time. The writer is convinced that this view is wholly mistaken. Nature, he suggests, will best respond to a logical and carefully thought-out questionaire; indeed, if we ask her a single question, she will often refuse to answer until some other topic has been discussed."

RA Fisher, quoted in Larsen & Marx (2001, p 633)

Slide 6 ANOVA & Factorial Designs

NOTES:

Mill's Cannon of the Difference

See my Appendix of statistical terms

J. S. Mill's (1843) fifth cannon of experimental enquiry (The cannon of difference) "Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation"

Kendall & Stuart (1979), and Fisher, find two major problems with basing an experimental or sampling design on Mill's 5th cannon: 1) the one-phenomenon (factor)-at-a-time approach does not work because it fails to account for interactions and 2) "We can never be quite sure that all the important, or even the most important, causal factors have been incorporated in the structure of the experiment. Some may be quite unknown; others although known, may wrongly be considered to be of minor importance and deliberately neglected. We always need to guard against the perversion of the inferences within an experiment by adventitious outside effects."

Slide 7 Mill's Cannon of the Difference

NOTES:

Case 13.1: Intertidal Seaweed Grazers

A randomized blocked ANOVA

- 3 grazers (L,f,F): Limpets, small **f**ish (f), large **F**ish (F)
- Experimental unit: square rock surface 1 m on a side
- 6 treatments: **LfF** (All grazers),**fF** (limpets excluded with caustic paint),**Lf** (coarse mesh), f (Limpets & Large fish excluded),**L** (fine mesh excludes fish),**C** Control (all grazers excluded)

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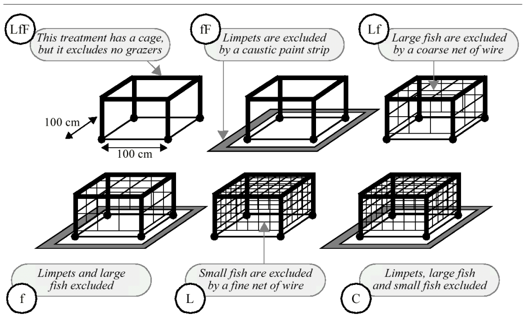
Slide 8 Case 13.1: Intertidal Seaweed Grazers

NOTES:

Display 13.1

Six treatments excluding three kinds of intertidal grazers from regenerating seaweed on the Oregon coast



Slide 9

NOTES:

Randomized Block ANOVA

Blocking increases the power of the test

- *Block 1: Just below the high tide level, exposed to heavy surf*
- *Block 2: Just below the high tide level, protected*
- Block 3: middle exposed
- Block 4: middle protected
- Block 6: just above low tide, exposed
- Block 7: On **near-vertical** rock wall, midtide protected
- Block 8: On **near-vertical** rock wall, above low tide, protected

Slide 10 Randomized Block ANOVA

NOTES:

Results of grazer study

Percentage of regenerated seaweed

Display 13.2

Percent cover by regenerating seaweed on plots with different grazers excluded, in eight blocks of differing tidal situation and exposure

	Treatment: Grazers with Access					
Block #	Control	L	f	Lf	fF	LfF
1	14 23	4 4	11 24	3 5	10 13	1 2
2	22 35	7 8	14 31	3 6	10 15	3 5
3	67 82	28 58	52 59	9 31	44 50	6 9
4	94 95	27 35	83 89	21 57	57 73	7 22
5	34 53	11 33	33 34	5 9	26 42	5 6
6	58 75	16 31	39 52	26 43	38 42	10 17
7	19 47	6 8	43 53	4 12	29 36	5 14
8	53 61	15 17	30 37	12 18	11 40	5 7

Slide 11 Results of grazer study

NOTES:

Results

A controlled experiment, but inference to a larger population (intertidal communities) may not be warranted

- Litte evidence for treatment differences among blocks
- Limpets have a very strong effect on seaweed (median regeneration with limpets only 16% of regeneration when they were excluded (95% CI 12.6 to 20.5%)
- Small and large fish, 54% (40.2%, 70.9%) and 68.5% (50.1%, 90.9%)
- No evidence that limpet effect depended on whether either large or small fish were present (p = 0.5 & 0.7 respectively)
- Can this study be used to make inferences to the entire population?

Slide 12 Results

NOTES:

Strategies

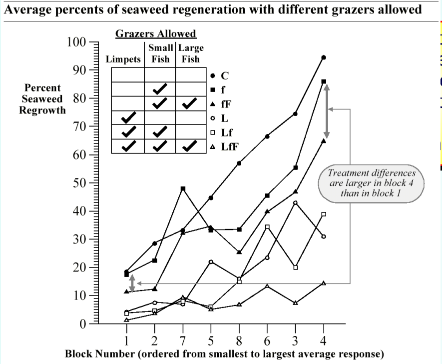
- Analyze the data graphically for outliers
- Fit the rich model, examine the residual plots to assess need for transformation of response variable
- With interactions, graphically display the data or use multiway tables
- Look at particular terms in the additive model to examine particular effects
- ANOVA F-test for additivity: interaction MS over error MS

Slide 13 Strategies

NOTES:

Display 13.7

Average percents of seaweed regeneration with different grazers allowed



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Slide 14

NOTES:

Significant Interaction term

with untransformed % Cover

Tests of Between-Subjects Effects

Dependent Variable: % Cover

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	**48763.5** [a]	**47**	**1037.5**	**11.6**	**1.1E-14**
Intercept	**78661.5**	**1**	**78661.5**	**880.3**	**1.6E-32**
BLOCK	**19105.5**	**7**	**2729.4**	**30.5**	**1.3E-15**
TREAT	**23045.5**	**5**	**4609.1**	**51.6**	**3.7E-18**
BLOCK * TREAT	**6612.5**	**35**	**188.9**	**2.1**	**.008**
Error	4289.0	48	89.4		
Total	131714.0	96			
Corrected Total	53052.5	95			

a. R Squared = .919 (Adjusted R Squared = .840)

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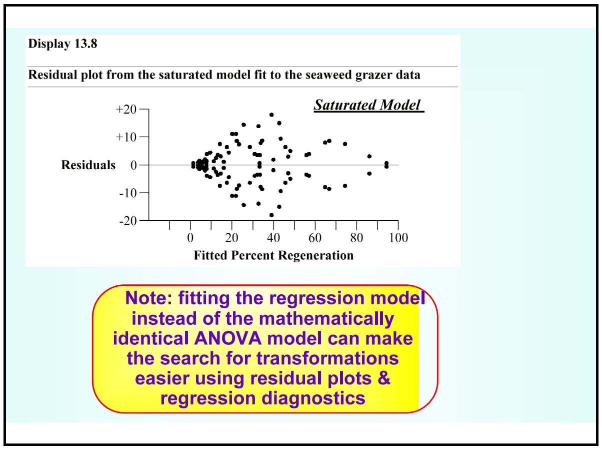
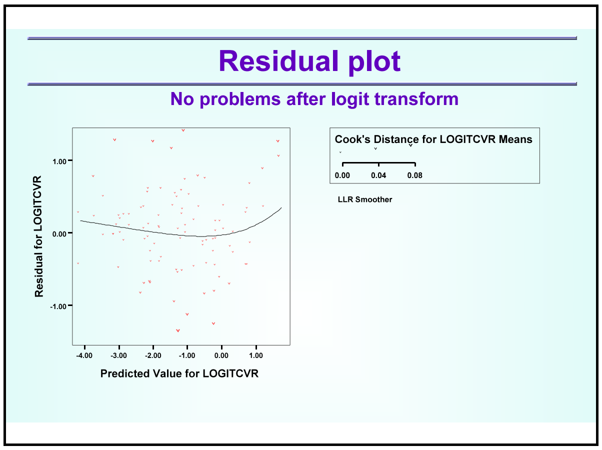
Slide 15 Significant Interaction term

NOTES:



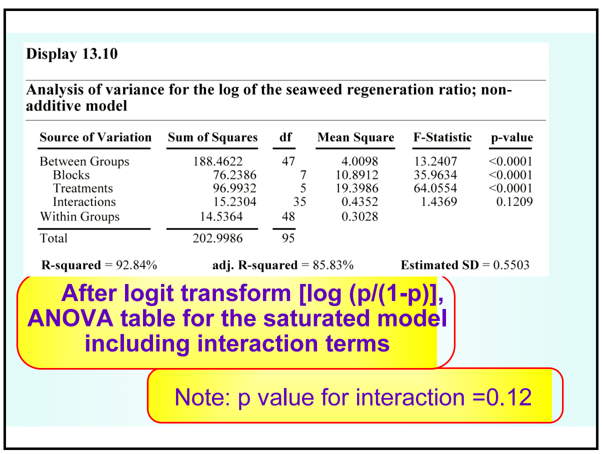
Slide 16 Fitting the regression model instead of the ANOVA model makes the search for transformations easier

NOTES:



Slide 17 Residual plot

NOTES:



Display 13.10

Analysis of variance for the log of the seaweed regeneration ratio; non-additive model

Source of Variation	Sum of Squares	df	Mean Square	F-Statistic	p-value
Between Groups	188.4622	47	4.0098	13.2407	<0.0001
Blocks	76.2386	7	10.8912	35.9634	<0.0001
Treatments	96.9932	5	19.3986	64.0554	<0.0001
Interactions	15.2304	35	0.4352	1.4369	0.1209
Within Groups	14.5364	48	0.3028		
Total	202.9986	95			

R-squared = 92.84% adj. R-squared = 85.83% Estimated SD = 0.5503

Slide 18 After logit transform [log (p/(1-p)]

ANOVA table for the saturated model, with interaction terms

NOTES:

The old syntax for fitting Factorial ANOVA

* Old ANOVA syntax, use GLM instead.

ANOVA logitcvr by block(1,8) treat(1,6).

ANOVA[a,b]

			Unique Method				
			Sum of Squares	df	Mean Square	F	Sig.
Logit(Cover)	Main Effects	(Combined)	**173.232**	**12**	**14.436**	**47.668**	**1.7E-22**
		Block	**76.239**	**7**	**10.891**	**35.963**	**5.4E-17**
		Treatment	**96.993**	**5**	**19.399**	**64.055**	**4.5E-20**
	2-Way Interactions	**Block * Treatment**	**15.230**	**35**	**.435**	**1.437**	**.121**
	Model		188.462	47	4.010	13.241	7.5E-16
	Residual		14.536	48	.303		
	Total		202.999	95	2.137		

a. Logit(Cover) by Block, Treatment

b. All effects entered simultaneously

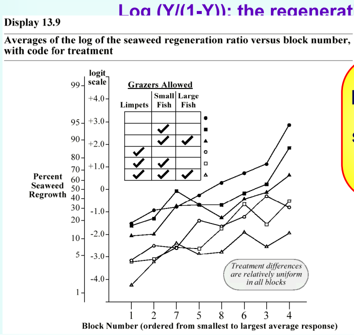
Note: p value for interaction =0.12

Slide 19 The old syntax for fitting Factorial ANOVA

NOTES:

After logit transform

Log (Y/(1-Y)): the regeneration ratio



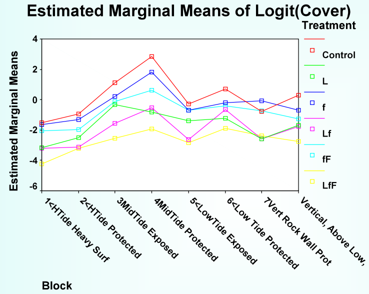
Note nearly parallel slopes, the key signature of no interaction among effects

Slide 20 After logit transform

NOTES:

Expected values, saturated model

The 35 interaction terms don't explain a significant additional amount of the variation in the data



Slide 21 Expected values, saturated model

NOTES:

Display 13.11 p. 375

Analysis of variance for the log of the seaweed regeneration ratio; additive model

Source of Variation	Sum of Squares	df	Mean Square	F-Statistic	p-value
Model	173.2318	12	14.4360	40.2520	<0.0001
Blocks	76.2386	7	10.8912	30.3684	<0.0001
Treatments	96.9932	5	19.3986	54.0900	<0.0001
Residual	29.7668	83	0.35864		
Total	202.9986	95			

R-squared = 85.34% adj. R-squared = 83.22% Estimated SD = 0.5989

Interaction term pooled with residual sum of squares, to produce tests with error df=83 (but note higher MSE [0.36 vs. 0.30])

Slide 22 Interaction term pooled with residual sum of squares, to produce tests with error df=83 (but note higher MSE [0.36 vs. 0.30])

NOTES:

Pooling Interaction SS (1 of 3)

Neter *et al.* (1996) {applications of rules to 13.1 in red}

- Don't pool the interaction & error SS unless
  - (1) The degrees freedom for MSE is small, perhaps 5 or less *No*
  - (2) The test statistic MS_interaction / MS_error falls substantially below the action limit of the decision rule, perhaps MS_intxn/MS_error < 2 for α = 0.05. *Yes*
- (1) assures that there will be increased power from pooling and (2) is to minimize the probability of Type II error for the interaction effect.
- **Conclusion for Case Study 13.1: Don't pool**

Source of Variation	Sum of Squares	df	Mean Square	F-Statistic	p-value
Between Groups	188.4622	47	4.0098	13.2407	<0.0001
Blocks	76.2386	7	10.8912	35.9634	<0.0001
Treatments	96.9932	5	19.3986	64.0554	<0.0001
Interactions	15.2304	35	0.4352	1.4369	0.1209
Within Groups	14.5364	48	0.3028		
Total	202.9986	95			

Slide 23 Pooling Interaction SS (1 of 3)

NOTES:

When can & should you pool interaction MS with error MS?

Pooling interaction terms (Slide 2 of 3)

- Underwood (1997, p. 273) discusses pooling with nested models, but the same argument would apply here
  - Test for the interaction MS effect with α = 0.05
    - If the test is significant, never pool.
    - If the test is not significant:
      - If the p value is > 0.25, pool
      - **If the p value is < 0.25, don't pool**

Source of Variation	Sum of Squares	df	Mean Square	F-Statistic	p-value
Between Groups	188.4622	47	4.0098	13.2407	<0.0001
Blocks	76.2386	7	10.8912	35.9634	<0.0001
Treatments	96.9932	5	19.3986	64.0554	<0.0001
Interactions	15.2304	35	0.4352	1.4369	0.1209
Within Groups	14.5364	48	0.3028		
Total	202.9986	95			

Slide 24 When can & should you pool interaction MS with error MS?

NOTES:

Pooling Interaction SS (Slide 3 of 3)

Quinn & Keough (2002,p. 260)

- Most statistics texts follow a 'sometimes pool' strategy
  - Underwood (1997) & Winer et al. (1991):
    - Pool if p value > 0.25;
    - Hayes: pool only if p value > 0.5
  - Sokal & Rohlf: p > 0.25 or 0.5
  - Quinn & Keough (2002): p value > 0.25
- **Application to Case Study 13.1: Don't pool**

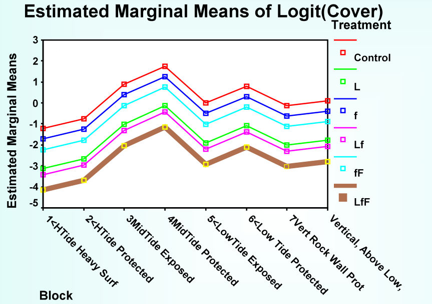
Source of Variation	Sum of Squares	df	Mean Square	F-Statistic	p-value
Between Groups	188.4622	47	4.0098	13.2407	<0.0001
Blocks	76.2386	7	10.8912	35.9634	<0.0001
Treatments	96.9932	5	19.3986	64.0554	<0.0001
Interactions	15.2304	35	0.4352	1.4369	0.1209
Within Groups	14.5364	48	0.3028		
Total	202.9986	95			

Slide 25 Pooling Interaction SS (Slide 3 of 3)

NOTES:

Expected logit (% Cover)

The pattern if interaction terms pooled



Slide 26 Expected logit (% Cover)

NOTES:

Expected effects

This display would have to be redone if interaction terms left in the model. The interpretation would be considerably more complex

Display 13.12

Table of averages of log percent seaweed regeneration ratio with different grazer combinations in eight blocks

Block	Treatment: Grazers with Access: Control	L	f	Lf	fF	LfF	Block Average	Block Effect
1	-1.51	-3.18	-1.62	-3.21	-2.05	-4.24	-2.64	-1.40
2	-0.94	-2.51	-1.31	-3.11	-1.97	-3.21	-2.18	-0.94
3	1.11	-0.31	0.22	-1.56	-0.12	-2.53	-0.53	0.70
4	2.85	-0.81	1.84	-0.52	0.64	-1.93	0.34	1.58
5	-0.27	-1.40	-0.69	-2.63	-0.68	-2.83	-1.42	-0.19
6	0.71	-1.23	-0.18	-0.66	-0.41	-1.89	-0.61	0.62
7	-0.79	-2.60	-0.08	-2.59	-0.74	-2.38	-1.53	-0.29
8	0.28	-1.66	-0.64	-1.75	-1.25	-2.77	-1.31	-0.07
Treatment Average	0.18	-1.71	-0.31	-2.00	-0.82	-2.72	-1.23	
Treatment Effect	1.41	-0.48	0.92	-0.77	0.41	-1.49		

Slide 27 Expected effects

NOTES:

SPSS Descriptive statistics

Use linear contrasts to find effect of individual grazers

1. Block

Dependent Variable: Logit(Cover)

Block	Mean	Std. Error	95% Confidence Interval Lower Bound	Upper Bound
1<HTide Heavy Surf	-2.6	.17	-3.0	-2.3
2<HTide Protected	-2.2	.17	-2.5	-1.8
3MidTide Exposed	-.5	.17	-.9	-.2
4MidTide Protected	.345	.173	.001	.689
5<LowTide Exposed	-1.4	.17	-1.8	-1.1
6<Low Tide Protected	-.6	.17	-1.0	-.3
7Vert Rock Wall Protected	-1.5	.17	-1.9	-1.2
Vertical, Above Low, Protected	-1.3	.17	-1.6	-1.0

Block Average	Block Effect
-2.64	-1.40
-2.18	-0.94
-0.53	0.70
0.34	1.58
-1.42	-0.19
-0.61	0.62
-1.53	-0.29
-1.31	-0.07
-1.23	

2. Treatment

Dependent Variable: Logit(Cover)

Treatment	Mean	Std. Error	95% Confidence Interval Lower Bound	Upper Bound
Control	.2	.15	-.1	.5
L	-1.7	.15	-2.0	-1.4
f	-.31	.15	-.61	-.02
Lf	-2.0	.15	-2.3	-1.7
fF	-.8	.15	-1.1	-.5
LfF	-2.7	.15	-3.0	-2.4

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Slide 28 SPSS Descriptive statistics

NOTES:

Linear contrasts

Used to test the effects of individual grzers from factorial design

Display 13.13

Separate effects of grazers using linear combinations of treatment means

Treatment:	LfF	fF	Lf	f	L	C	Estimate	Standard Error	t-Stat
Sample size:	16	16	16	16	16	16			
Average:	-2.7247	-0.8214	-2.0044	-0.3137	-1.7120	+0.1805			
Large Fish:	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	-0.6140	0.1497	4.10
Small Fish:	0	0	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-0.3933	0.1497	2.63
Limpets:	$+\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{1}{3}$	$-\frac{1}{3}$	-1.8288	0.1222	14.97
Limpets x Small:	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	- 1	+ 1	+0.0955	0.2593	0.37
Limpets x Large:	+ 1	- 1	- 1	+ 1	0	0	-0.2126	0.2994	0.71

(Contrast Summary: Estimate, Standard Error, t-Stat)

Slide 29 Linear contrasts

NOTES:

Large Fish Effect

Large Fish Effect: {LfF(6) -Lf (4)} + {fF(5)- f(3)}

```
* Treatment levels in data file.
* LfF=6, fF=5, Lf=4,f=3,L=2, C=1.
UNIANOVA
logitcvr  BY block treat
/LMatrix = "Large fish"
treat 0 0 -1/2 -1/2 1/2 1/2
/LMatrix = "Small fish"
treat -1/2 -1/2 1/2 1/2 0 0
/LMatrix = "Limpets"
treat -1/3 1/3 -1/3 1/3 -1/3 1/3
/LMatrix = "Limpets x small"
treat  1 -1 -1/2 1/2 -1/2 1/2
/LMatrix = "Limpets x Large"
treat  0 0 1 -1 -1 1
/METHOD = SSTYPE(3)
/INTERCEPT = INCLUDE
/PLOT = PROFILE( block*treat )
/CRITERIA = ALPHA(.05)
/DESIGN = block treat .
```

Contrast Results (K Matrix)[a]

Contrast			Dependent Variable Logit(Cover)
L1	Contrast Estimate		-.6
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		-.6
	Std. Error		.15
	Sig.		1.E-04
	95% Confidence Interval for Difference	Lower Bound	-.9
		Upper Bound	-.3

a. Based on the user-specified contrast coefficients (L') matrix: Large fish

Slide 30 Large Fish Effect

NOTES:

Small Fish Effect

Small Fish Effect: {Lf(4) -L (2)} + {f(3) - Control (1)}

```
* Treatment levels in data file.
* LfF=6, fF=5, Lf=4,f=3,L=2, C=1.
UNIANOVA
logitcvr  BY block treat
/LMatrix = "Large fish"
treat 0 0 -1/2 -1/2 1/2 1/2
/LMatrix = "Small fish"
treat -1/2 -1/2 1/2 1/2 0 0
/LMatrix = "Limpets"
treat -1/3 1/3 -1/3 1/3 -1/3 1/3
/LMatrix = "Limpets x small"
treat  1 -1 -1/2 1/2 -1/2 1/2
/LMatrix = "Limpets x Large"
treat  0 0 1 -1 -1 1
/METHOD = SSTYPE(3)
/INTERCEPT = INCLUDE
/PLOT = PROFILE( block*treat )
/CRITERIA = ALPHA(.05)
/DESIGN = block treat .
```

Contrast Results (K Matrix)[a]

Contrast			Dependent Variable Logit(Cover)
L1	Contrast Estimate		-.4
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		-.4
	Std. Error		.15
	Sig.		.01
	95% Confidence Interval for Difference	Lower Bound	-.7
		Upper Bound	-.1

a. Based on the user-specified contrast coefficients (L') matrix: Small fish

Slide 31 Small Fish Effect

NOTES:

Limpet Effect

Limpet Effect:{LfF(6) - fF (5)} + {Lf(4) - f (3)} +{L(2)-Control (1)}

```
* Treatment levels in data file.
* LfF=6, fF=5, Lf=4,f=3,L=2, C=1.
UNIANOVA
logitcvr  BY block treat
/LMatrix = "Large fish"
treat 0 0 -1/2 -1/2 1/2 1/2
/LMatrix = "Small fish"
treat -1/2 -1/2 1/2 1/2 0 0
/LMatrix = "Limpets"
treat -1/3 1/3 -1/3 1/3 -1/3 1/3
/LMatrix = "Limpets x small"
treat  1 -1 -1/2 1/2 -1/2 1/2
/LMatrix = "Limpets x Large"
treat  0 0 1 -1 -1 1
/METHOD = SSTYPE(3)
/INTERCEPT = INCLUDE
/PLOT = PROFILE( block*treat )
/CRITERIA = ALPHA(.05)
/DESIGN = block treat .
```

Contrast Results (K Matrix)[a]

Contrast			Dependent Variable Logit(Cover)
L1	Contrast Estimate		-1.8
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		-1.8
	Std. Error		.122
	Sig.		.000
	95% Confidence Interval for Difference	Lower Bound	-2.1
		Upper Bound	-1.6

a. Based on the user-specified contrast coefficients (L') matrix: Limpets

Slide 32 Limpet Effect

NOTES:

Conclusions about Case 13.1

- Pooling interactions simplifies the interpretation of the results, but at the expense of violating accepted practice
  - P values > 0.05 doesn't mean that there are no interaction effects
  - Block interaction effects should be assessed
  - This could result in rejection of the paper
- A fixed effects model was used, restricting inferences to these areas. A random effects model might have been better.
- Logistic regression (Ch 21) with a binomial response might be an alternative model for analyzing % cover data
  - Bob Miller (UMB Biology Ph.D, 2005) analyzed grazers and found 0% algal cover in many plots. Parametric ANOVA could not be used.

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Slide 33 Conclusions about Case 13.1

NOTES:

Case 13.2 Pygmalion Effect

Slide 34 Case 13.2 Pygmalion Effect

NOTES:

Pygmalion effect

A study to avoid interpersonal interactions

- Tracking in schools:
  - Good students get better and poor students get worse
  - Self-fulfilling prophecies
- Goal of the study by Dov Eden: Pygmalion without interpersonal contrast effects
- Ten companies selected (9 in data), 3 platoons in each company, 1 platoon leader out of 3 told he had an exceptional group

Slide 35 Pygmalion effect

NOTES:

Pygmalion Effect

Mean scores for the platoons to be contrasted

Display 13.3

Average scores of soldiers on the Practical Specialty Test, for platoons given the Pygmalion treatment and for control platoons

	Treatments		
Company	*Pygmalion*	*Control*	
1	80.0	63.2	69.2
2	83.9	63.1	81.5
3	68.2	76.2	
4	76.5	59.5	73.5
5	87.8	73.9	78.5
6	89.8	78.9	84.7
7	76.1	60.6	69.6
8	71.5	67.8	73.2
9	69.5	72.3	73.9
10	83.7	63.7	77.7

Slide 36 Pygmalion Effect

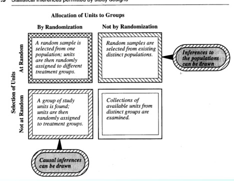
NOTES:

Pygmalion results

Note: addition of random effects model

- **Pygmalion treatment added 7.2 (±5.4) points to a platoon's score**
- **Very strong evidence that the Pygmalion effect is real (Fixed effect, randomized block ANOVA, $F_{1,18}$=7.8; 1-sided p = 0.006)**
- **Because of the randomized design, a causal inference can be made for this group of 10 companies**
- **If these companies are representative of all army companies, there is strong evidence that the effect would be found throughout Army companies (Mixed model ANOVA, $F_{1,9.2}$=8.7; 1-sided p = 0.008)**

Display 1.5 Statistical inferences permitted by study designs



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Slide 37 Pygmalion results

NOTES:

Strategies for factorial analysis

- *Decide at the design stage whether factors are fixed or random*
- Analyze the data graphically for outliers, need for transformation
- Fit the rich model (saturated model) examine the residual plots
- With interactions, graphically display the data or use multiway tables
- Look at particular terms in the additive model to examine particular effects
- ANOVA F-test for additivity, Interaction MS over error MS
  - Use appropriate rules for pooling:
  - Pool only if p>0.25 and only if df for MSE is < 5
- *Test main effects over appropriate error term for fixed or random effects model*

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Slide 38 Strategies for factorial analysis

NOTES:

Additive and non-additive models

- Both Ch 13 Case Studies can be viewed as additive models
  - 13.1 Area + predator effects (no intxn)
  - 13.2: Block (Company) + Pygmalion effect
- Additive model: both block and factor add fixed amount

Most recent statistics texts, esp. in ecology, accept the reduced (additive) model if the interaction p values > 0.25 or 0.5

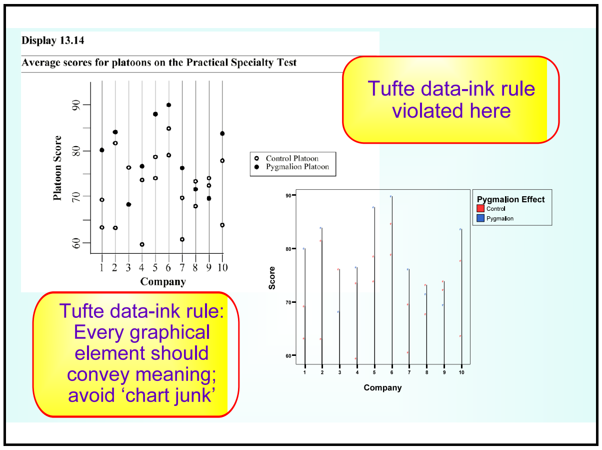
Display 13.4

Hypothetical mean scores on the Practical Specialty Test, illustrating additivity of treatment and company effects

	Treatments		Treatment Effects
Company	*Pygmalion*	*Control*	*(Pygmalion - Control)*
1	78	70	8
2	85	77	8
3	79	71	8
4	72	64	8
5	84	76	8
6	89	81	8
7	73	65	8
8	76	68	8
9	75	67	8
10	82	74	8

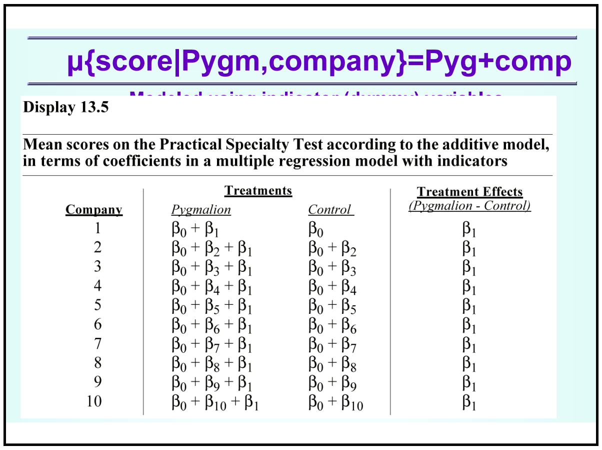
Slide 39 Additive and non-additive models

NOTES:



Slide 40

NOTES:



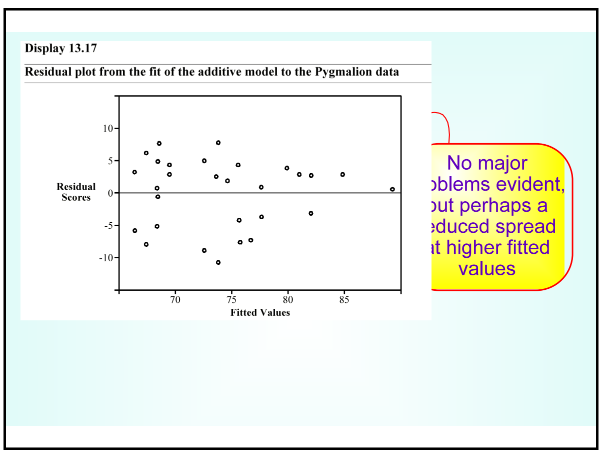
Display 13.5

Mean scores on the Practical Specialty Test according to the additive model, in terms of coefficients in a multiple regression model with indicators

Company	Treatments: Pygmalion	Treatments: Control	Treatment Effects (Pygmalion - Control)
1	$\beta_0 + \beta_1$	β_0	β_1
2	$\beta_0 + \beta_2 + \beta_1$	$\beta_0 + \beta_2$	β_1
3	$\beta_0 + \beta_3 + \beta_1$	$\beta_0 + \beta_3$	β_1
4	$\beta_0 + \beta_4 + \beta_1$	$\beta_0 + \beta_4$	β_1
5	$\beta_0 + \beta_5 + \beta_1$	$\beta_0 + \beta_5$	β_1
6	$\beta_0 + \beta_6 + \beta_1$	$\beta_0 + \beta_6$	β_1
7	$\beta_0 + \beta_7 + \beta_1$	$\beta_0 + \beta_7$	β_1
8	$\beta_0 + \beta_8 + \beta_1$	$\beta_0 + \beta_8$	β_1
9	$\beta_0 + \beta_9 + \beta_1$	$\beta_0 + \beta_9$	β_1
10	$\beta_0 + \beta_{10} + \beta_1$	$\beta_0 + \beta_{10}$	β_1

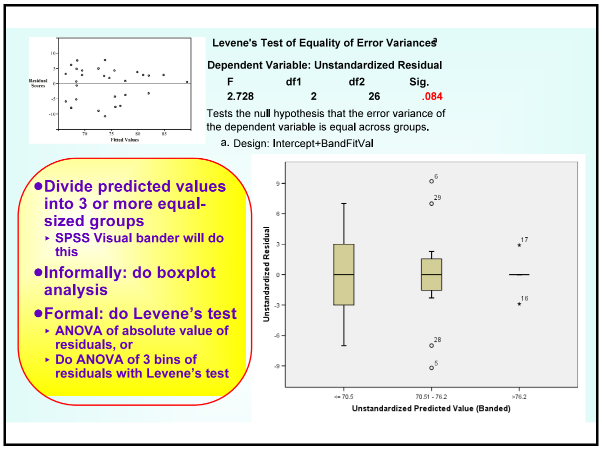
Slide 41 μ{score|Pygm,company}=Pyg+comp

NOTES:



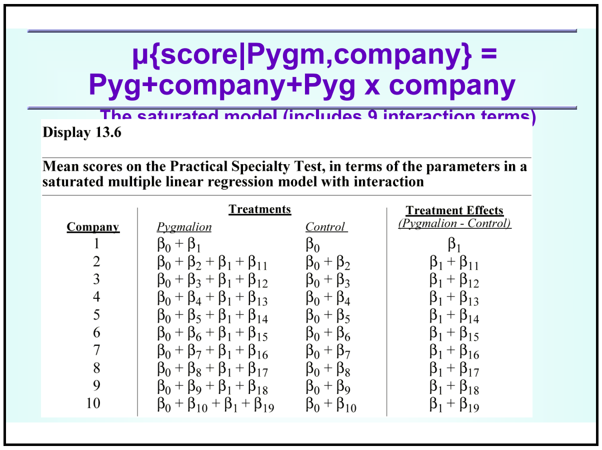
Slide 42

NOTES:



Slide 43

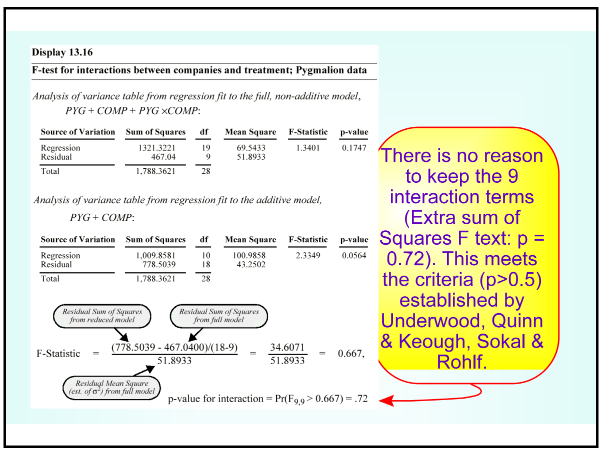
NOTES:



	Treatments		Treatment Effects
Company	Pygmalion	Control	(Pygmalion - Control)
1	$\beta_0 + \beta_1$	β_0	β_1
2	$\beta_0 + \beta_2 + \beta_1 + \beta_{11}$	$\beta_0 + \beta_2$	$\beta_1 + \beta_{11}$
3	$\beta_0 + \beta_3 + \beta_1 + \beta_{12}$	$\beta_0 + \beta_3$	$\beta_1 + \beta_{12}$
4	$\beta_0 + \beta_4 + \beta_1 + \beta_{13}$	$\beta_0 + \beta_4$	$\beta_1 + \beta_{13}$
5	$\beta_0 + \beta_5 + \beta_1 + \beta_{14}$	$\beta_0 + \beta_5$	$\beta_1 + \beta_{14}$
6	$\beta_0 + \beta_6 + \beta_1 + \beta_{15}$	$\beta_0 + \beta_6$	$\beta_1 + \beta_{15}$
7	$\beta_0 + \beta_7 + \beta_1 + \beta_{16}$	$\beta_0 + \beta_7$	$\beta_1 + \beta_{16}$
8	$\beta_0 + \beta_8 + \beta_1 + \beta_{17}$	$\beta_0 + \beta_8$	$\beta_1 + \beta_{17}$
9	$\beta_0 + \beta_9 + \beta_1 + \beta_{18}$	$\beta_0 + \beta_9$	$\beta_1 + \beta_{18}$
10	$\beta_0 + \beta_{10} + \beta_1 + \beta_{19}$	$\beta_0 + \beta_{10}$	$\beta_1 + \beta_{19}$

Slide 44 μ{score|Pygm,company} = Pyg+company+Pyg x company

NOTES:



Source of Variation	Sum of Squares	df	Mean Square	F-Statistic	p-value
Regression	1321.3221	19	69.5433	1.3401	0.1747
Residual	467.04	9	51.8933		
Total	1,788.3621	28			

Source of Variation	Sum of Squares	df	Mean Square	F-Statistic	p-value
Regression	1,009.8581	10	100.9858	2.3349	0.0564
Residual	778.5039	18	43.2502		
Total	1,788.3621	28			

Slide 45

NOTES:

Extra sum of squares F test

Enter 3 models hiearchically using /Analyze/Regression
The 9 interaction terms do not explain a significant portion of the residual variation.

Model Summary[d]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics: R Square Change	F Change	df1	df2	Sig. F Change
1	.428[a]	.183	.153	7.3561	.183	6.049	1	27	.021
2	.751[b]	.565	.323	6.5765	.382	1.753	9	18	.148
3	.860[c]	.739	.188	7.2037	.174	.667	9	9	.722

a. Predictors: (Constant), Pyg
b. Predictors: (Constant), Pyg, CMP10, CMP9, CMP3, CMP8, CMP7, CMP6, CMP5, CMP2, CMP4
c. Predictors: (Constant), Pyg, CMP10, CMP9, CMP3, CMP8, CMP7, CMP6, CMP5, CMP2, CMP4, INT9, INT8, INT6, INT5, INT2, INT7, INT4, INT10, INT3
d. Dependent Variable: Score

The 9 block x interaction terms, with a p value of 0.72 can be dropped

Slide 46 Extra sum of squares F test

NOTES:

Display 13.18

Multiple linear regression output from the fit of the additive model to the Pygmalion data: $\mu\{score \mid PYG, COMP\} = PYG + COMPANY$

Variable	Coefficient	Standard Error	t-Statistic	2-Sided p-Value
CONSTANT	75.6137	4.1682	18.1405	<.0001
pyg	**7.2205**	**2.5795**	**2.7992**	**.0119**
cmp2	5.3667	5.3697	0.9994	.3308
cmp3	0.1966	6.0189	0.0327	.9743
cmp4	-0.9667	5.3697	-0.1800	.8591
cmp5	9.2667	5.3697	1.7257	.1015
cmp6	13.6667	5.3697	2.5452	.0203
cmp7	-2.0333	5.3697	-0.3787	.7094
cmp8	0.0333	5.3697	0.0062	.9951
cmp9	1.1000	5.3697	0.2049	.8400
cmp10	4.2333	5.3697	0.7884	.4407

Estimated SD = 6.576 on 18 d.f.

The Pygmalion effect adds 7.2 (± 5.4) to the score of the typical platoon

Slide 47

NOTES:

Unbalanced designs & effect sizes

Different estimates of the treatment effect, from each company and from the combined data ignoring company differences

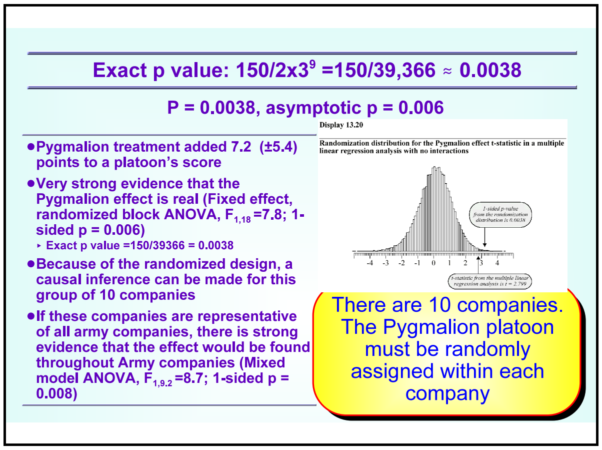
	i	Averages: Pygmalion	Control	Difference $\hat{\delta}_i$
Company	1	80.0	66.2	13.8
	2	83.9	72.3	11.6
	3	68.2	76.2	-8.0
	4	76.5	66.5	10.0
	5	87.8	76.2	11.6
	6	89.8	81.8	8.0
	7	76.1	65.1	11.0
	8	71.5	70.5	1.0
	9	69.5	73.1	-3.6
	10	83.7	70.7	13.0
	All	78.7000	71.6316	7.0684

Taking into
:count company
fects, effect size
7.22 (not 7.07)
d standard error
of estimate is
lower.

...the multiple linear regression estimate (of SD) will always give the most efficient weighting to estimates from different levels of a confounding variable in unbalanced situations. Sleuth p. 397

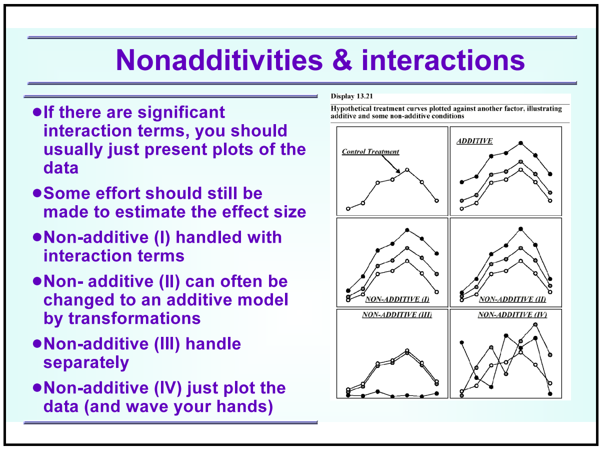
Slide 48 Unbalanced designs & effect sizes

NOTES:



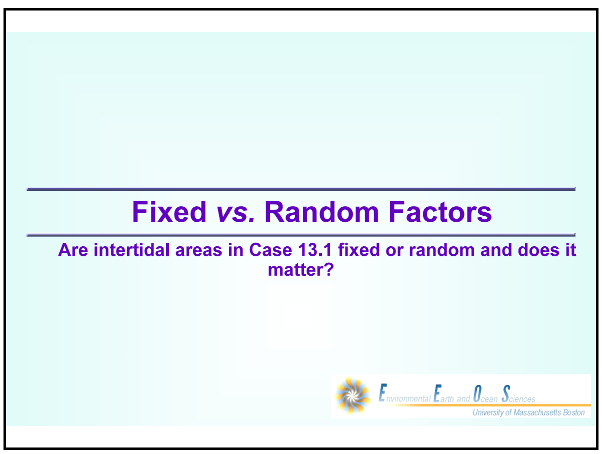
Slide 49 Exact p value: 150/2x39 =150/39,366 ≈ 0.0038

NOTES:



Slide 50 Nonadditivities & interactions

NOTES:



Slide 51 Fixed vs. Random Factors

NOTES:

Fixed vs. Random factors

$X_{ij} = \mu + A_i + e_{ij}$

where X_{ij} is jth replicate in ith treatment (ith level of factor A; $i = 1 \ldots a$), A_i is difference between ith level of factor A and overall mean of all levels (μ), e_{ij} is the deviation of replicate j in ith sample from the mean of that population.

Fixed factor:
By definition:

$$\sum_{i=1}^{a} A_i = 0$$

(see Section 7.6).

One way ANOVA
Fixed factor

Analysis of variance	Mean square estimates
Among treatments	$\sigma_e^2 + \frac{n\sum_{i=1}^{a}(A_i - \bar{A})^2}{(a-1)}$ or $\sigma_e^2 + nk_A^2$
Within treatments	σ_e^2

where k_A^2 indicates fixed differences, all sampled in the experiment. ✓

Slide 52 Fixed vs. Random factors

NOTES:

Fixed *vs.* Random factors

Tables from Underwood (1997)

Random factor:

$$E\left(\sum_{i=1}^{a} A_i\right) = 0$$

One way ANOVA
Random factor

Meaning you *expect* $\sum_{i=1}^{a} A_i = 0$ on average, over many experiments, but in a single experiment, A_i values as sampled may not sum to zero.

Analysis of variance	Mean square estimates
Among treatments	$\sigma_e^2 + n\sigma_A^2$
Within treatments	σ_e^2

where σ_A^2 is the variance of the population of A_i values sampled in your experiment.

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Slide 53 Fixed vs. Random factors

NOTES:

Model I Factorial ANOVA

Both factors fixed, from Underwood (1997)
Use Residual mean square as F statistic denominator to test main effects

(a) Both factors fixed

Source of variation	Sum of squares	Degrees of freedom	Mean square estimates	*F*-ratio versus
Among levels of A = A	$(a-1)\sigma_e^2 + bn\sum_{i=1}^{a}(A_i - \bar{A})^2$	$a-1$	$\sigma_e^2 + bnk_A^2$	Residual
Among levels of B = B	$(b-1)\sigma_e^2 + an\sum_{j=1}^{b}(B_j - \bar{B})^2$	$b-1$	$\sigma_e^2 + ank_B^2$	Residual
A × B	$(a-1)(b-1)\sigma_e^2 + n\sum_{i=1}^{a}\sum_{j=1}^{b}(AB_{ij} - \overline{AB}_i - \overline{AB}_j + \overline{AB})^2$	$(a-1)(b-1)$	$\sigma_e^2 + nk_{AB}^2$	Residual
Residual	$ab(n-1)\sigma_e^2$	$ab(n-1)$	σ_e^2	

Tests for difference in means

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Slide 54 Model I Factorial ANOVA

NOTES:

Model II Factorial ANOVA

Both factors random;
Both main effects tested using Interaction mean square in the denominator of the F statistic

(c) Both factors random

Source of variation	Sum of squares	Degrees of freedom	Mean square estimates	F-ratio versus
Among levels of A = A	$(a-1)\sigma_e^2 + (a-1)n\sigma_{AB}^2 + (a-1)bn\sigma_A^2$	$a-1$	$\sigma_e^2 + n\sigma_{AB}^2 + bn\sigma_A^2$	A × B
Among levels of B = B	$(b-1)\sigma_e^2 + (b-1)n\sigma_{AB}^2 + (b-1)an\sigma_A^2$	$b-1$	$\sigma_e^2 + n\sigma_{AB}^2 + an\sigma_B^2$	A × B
A × B	$(a-1)(b-1)\sigma_e^2 + (a-1)(b-1)n\sigma_{AB}^2$	$(a-1)(b-1)$	$\sigma_e^2 + n\sigma_{AB}^2$	Residual
Residual	$ab(n-1)\sigma_e^2$	$ab(n-1)$	σ_e^2	

Tests for difference in variances

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Slide 55 Model II Factorial ANOVA

NOTES:

Model II & Mixed Model (Model III) Factorial ANOVAs

Model III: At least 1 Fixed & 1 random factor
Test Fixed factor main effect vs. Interaction mean square, not error mean square

(b) A fixed, B random

Source of variation	Sum of squares	Degrees of freedom	Mean square estimates	F-ratio versus
Among levels of A = A	$(a-1)\sigma_e^2 + (a-1)n\sigma_{AB}^2 + bn\sum_{i=1}^{a}(A_i - \bar{A})^2$	$(a-1)$	$\sigma_e^2 + n\sigma_{AB}^2 + bnk_A^2$	$A \times B$
Among levels of B = B	$(b-1)\sigma_e^2 + (b-1)an\sigma_B^2$	$(b-1)$	$\sigma_e^2 + an\sigma_B^2$	Residual
A × B	$(a-1)(b-1)\sigma_e^2 + (a-1)(b-1)n\sigma_{AB}^2$	$(a-1)(b-1)$	$\sigma_e^2 + n\sigma_{AB}^2$	Residual
Residual	$ab(n-1)\sigma_e^2$	$ab(n-1)$	σ_e^2	

Tests for difference in means of A, after assessing the increase in variance due to the random factor B

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Slide 56 Model II & Mixed Model (Model III) Factorial ANOVAs

NOTES:

SPSS mixed effects ANOVA

If companies were randomly selected; Use if inferences are to be made to a larger population

```
UNIANOVA
score  BY pyg company
/RANDOM = company
/CONTRAST (pyg)=Simple
/METHOD = SSTYPE(3)
/INTERCEPT = INCLUDE
/PLOT = PROFILE( pyg*company )
/EMMEANS = TABLES(pyg) COMPARE AD
/PLOT = SPREADLEVEL RESIDUALS
/CRITERIA = ALPHA(.05)
/DESIGN = pyg company pyg*company.
```

If the interaction terms (pyg *company) are not included in the model, then the mixed effects ANOVA is identical to the fixed effects ANOVA

Slide 57 SPSS mixed effects ANOVA

NOTES:

When should a factor be regarded as random instead of fixed?

- Winer *et al.* (1991)
  - If the number of levels of a factor, *p*, is a very small fraction of the number of possible levels of a factor ($P_{effective}$), $p/P_{effective} \approx 0$ and the factor should be regarded as random
  - If the number of levels of a factor *p* is a large fraction of the total number of possible levels, then $p/P_{effective} \approx 1$ and the factor should be regarded as fixed
  - If the levels are random samples of the possible levels, then the factor should be considered random.

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Slide 58 When should a factor be regarded as random instead of fixed?

NOTES:

13.2 Companies as a random effect

Test Pygmalion main effect over interaction Mean Square
p value increased from 0.012 to 0.016

Tests of Between-Subjects Effects

Dependent Variable: Score

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	Hypothesis	146247.185	1	146247.185	1981.329	<.0000001
	Error	670.688	9.086	73.813[a]		
pyg	Hypothesis	301.843	1	301.843	8.692	**.016**
	Error	318.928	**9.185**	**34.724**[b]		
company	Hypothesis	665.663	9	73.963	2.137	.137
	Error	311.464	9	34.607[c]		
pyg * company	Hypothesis	311.464	9	34.607	.667	.722
	Error	467.040	9	51.893[d]		

a. .993 MS(company) + .007 MS(Error)
b. **.993 MS(pyg * company) + .007 MS(Error)**
c. MS(pyg * company)
d. MS(Error)

Non-integer *df* due to the unbalanced design

Slide 59 13.2 Companies as a random effect

NOTES:

Effect size of Pygmalion treatment

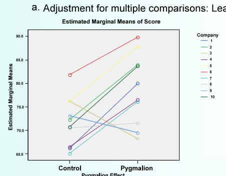
Note: two of the 10 groups declined

Pairwise Comparisons

Dependent Variable: Score

(I) Pygmalion Effect	(J) Pygmalion Effect	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a] Lower Bound	Upper Bound
Control	Pygmalion	**-6.840***	**2.836**	**.039**	**-13.256**	**-.424**
Pygmalion	Control	**6.840***	2.836	**.039**	.424	13.256

Based on estimated marginal means
*. The mean difference is significant at the .05 level.
a. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).



With the interaction term, the Pygmalion effect is reduced from 7.2 to 6.8 units [Identical effect in both fixed & mixed effects models]

Slide 60 Effect size of Pygmalion treatment

NOTES:

Conclusions to Case 13.2

- If the goal is to make inferences to all Army companies and platoons, then company should be treated as a random factor
  - The Pygmalion effect is tested vs. 'Pyg x company' interaction instead of error MS
  - The effect still offers evidence against the no-effect null hypothesis (p=0.008), but the p value is slightly larger than if a fixed effect model were used (p=0.006)

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Slide 61 Conclusions to Case 13.2

NOTES:

Conclusion from today's class

- Factorial ANOVA models are a subset of the general linear model
  - Can be analyzed using ANOVA, Regression, or GLM/Univariate
  - The results are mathematically identical
- Fisher noted that factorial ANOVA is superior to testing 1 factor at a time
- Interactions: factors have synergistic effects
  - Interactions must be assessed
    - Note that transforms can eliminate interaction effects
  - Pooling
    - Sleuth doesn't properly cover the problem of pooling interaction terms: use caution when pooling
    - Inappropriate pooling is an example of pseudoreplication & can give rise to Type II error (concluding no interaction or block effect when such effects exist)
    - At the least, use p>0.25 rule
- Random vs. Fixed factors in ANOVA designs
  - The choice should be made *a priori*
  - Interaction MS used as denominator to test main effects in Model II and Model III (mixed model) Factorial ANOVA

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Slide 62 Conclusion from today's class

NOTES: