

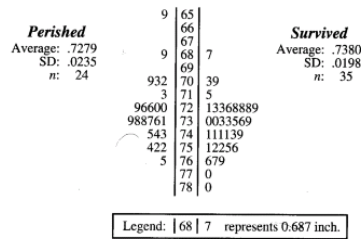
<p style="text-align: center;">Class 4, Chapter 2: Inferences using t- distributions</p> <p style="text-align: center;">Chapter 3: A Closer Look at Assumptions [of the <i>t</i> tests]</p> <p style="text-align: center;">2/9/09 M</p>	<p>Slide 1 Class 4, Chapter 2: Inferences using t-distributions</p> <p>Chapter 3: A Closer Look at Assumptions [of the <i>t</i> tests]</p> <p>NOTES:</p>
<p style="text-align: center;">HW 3 for Thus 2/12/09 11 am</p> <p style="text-align: center;">Submit as Myname-HW3.doc (or *.rtf)</p> <ul style="list-style-type: none"> • Finish Chapter 2 and start on Chapter 3 “A closer look at assumptions” <ul style="list-style-type: none"> ▸ Read Sterne & Smith (2001) “Sifting the evidence” [Discusses <i>p</i> values & significance testing] • Conceptual exercises, Chapter 2 <ul style="list-style-type: none"> ▸ Post ≥1 message & ≥1 reply to a message on the Blackboard Vista 4 discussion section. • Chapter 2 computation problems (SPSS data on Blackboard Vista 4) <ul style="list-style-type: none"> ▸ 2.21Bumpus’s data: weights of Bumpus’s birds 	<p>Slide 2 HW 3 for Thus 2/12/09 11 am</p> <p>NOTES:</p>
<p style="text-align: center;">HW 4 due Mon 2/16/09 9:50 am</p> <p style="text-align: center;">Submit as Myname-HW4.doc (or *.rtf)</p> <ul style="list-style-type: none"> • Finish Ch 3 for Weds’ class <ul style="list-style-type: none"> ▸ Chapter 3: A closer look at assumptions ▸ Read <ul style="list-style-type: none"> ■ Hayek & Buzas (1997, on sampling) ■ Hurlbert (1984) on Pseudoreplication ▸ Post one comment and one reply to issues raised in Hayek & Buzas or Hurlbert (1984) • Chapter 3 problem due Monday 2/16 <ul style="list-style-type: none"> ▸ 3.28 Pollen removal 	<p>Slide 3 HW 4 due Mon 2/16/09 9:50 am</p> <p>NOTES:</p>

<div data-bbox="347 170 665 205" data-label="Section-Header"> <h3>Student Presentations</h3> </div> <div data-bbox="253 245 646 386" data-label="List-Group"> <ul style="list-style-type: none"> • We'll cover statistical examples on Weds. • Angeliki or I will notify the presenters on Weds, but you can start preparations now </div> <div data-bbox="652 508 779 539" data-label="Text"> <p>EEOS611</p> </div>	<div data-bbox="815 132 1235 170" data-label="Section-Header"> <h3>Slide 4 Student Presentations</h3> </div> <div data-bbox="815 256 941 291" data-label="Text"> <p>NOTES:</p> </div>
<div data-bbox="362 762 649 806" data-label="Section-Header"> <h3>Sleuth Chapter 2</h3> </div> <div data-bbox="358 812 662 842" data-label="Text"> <p>Inference using t-distributions</p> </div>	<div data-bbox="815 621 1169 661" data-label="Section-Header"> <h3>Slide 5 Sleuth Chapter 2</h3> </div> <div data-bbox="815 804 941 840" data-label="Text"> <p>NOTES:</p> </div>
<div data-bbox="272 1146 751 1182" data-label="Section-Header"> <h3>Weiner's account of Bumpus data</h3> </div> <div data-bbox="274 1186 750 1234" data-label="Text"> <p>1994. The beak of the finch: a story of evolution in our time. Alfred A. Knopf, New York.</p> </div> <div data-bbox="250 1241 570 1495" data-label="List-Group"> <ul style="list-style-type: none"> • English sparrows had been introduced in New York's Central Park in 1851. An eccentric bird lover wanted to import every one of the birds in Shakespeare's plays to the United States. "So the birds were lying in the snow that morning in part because Shakespeare had written, 'There is a special providence in the fall of a sparrow.' • Last day of January 1898, huge storm, large number of English sparrows lay dead </div> <div data-bbox="625 1312 725 1472" data-label="Image"> </div> <div data-bbox="652 1482 779 1514" data-label="Text"> <p>EEOS611</p> </div>	<div data-bbox="815 1108 1406 1148" data-label="Section-Header"> <h3>Slide 6 Weiner's account of Bumpus data</h3> </div> <div data-bbox="815 1232 941 1266" data-label="Text"> <p>NOTES:</p> </div>

Bumpus sparrow data

Stem-and-leaf plot

Display 2.1 Humerus lengths (inches) of adult male house sparrows, 24 that perished and 35 that survived in a winter storm



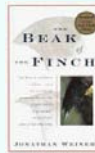
Slide 7 Bumpus sparrow data

NOTES:

Bumpus's sparrow data

From Weiner (1994, p. 227-228) "The Beak of the Finch"

- In the early 1970s, Peter Grant reanalyzed Bumpus's data, "He concluded that Bumpus had actually seen not one but two kinds of natural selection. For the female sparrows the storm was stabilizing. The event killed the largest and the smallest but preserved the mean, just as Bumpus had said. In the males, however, the pressure of the storm was directional, pushing the birds toward **smaller** size. The reanalysis of Bumpus's classic data helped inspire the Grants' first trip to the Galapagos."



Slide 8 Bumpus's sparrow data

NOTES:

Anatomical abnormalities & schizophrenia

Case 2.2: 15 pairs of twins, paired *t* test

Display 2.2

Differences in volumes (cm^3) of left hippocampus in fifteen sets of monozygotic twins where one twin is affected by schizophrenia

Pair #	Unaffected	Affected	Difference	Differences
1	1.94	1.27	0.67	-2
2	1.44	1.63	-0.19	-1
3	1.56	1.47	0.09	0
4	1.58	1.39	0.19	1
5	2.06	1.93	0.13	2
6	1.66	1.26	0.40	3
7	1.75	1.71	0.04	4
8	1.77	1.67	0.10	5
9	1.78	1.28	0.50	6
10	1.92	1.85	0.07	7
11	1.25	1.02	0.23	
12	1.93	1.34	0.59	
13	2.04	2.02	0.02	
14	1.62	1.59	0.03	
15	2.08	1.97	0.11	

Average: 0.199
Sample SD: 0.238
n: 15

Legend: | 6 | 7 represents 0.67 cm^3

Slide 9 Anatomical abnormalities & schizophrenia

NOTES:

Case 2.2 Statistical Summary

Sleuth, p. 31

There is substantial evidence that the mean difference in the left hippocampus volumes between schizophrenic individuals and their nonschizophrenic twins is nonzero (two-sided p -value = 0.006, from a paired t test). It is estimated that the mean volume is 0.20 cm^3 smaller for those with schizophrenia (about 11% smaller). A 95% confidence interval for the difference is from 0.07 to 0.33 cm^3 .

Slide 10 Case 2.2 Statistical Summary

NOTES:

Statistical Summary includes elements of Fisher, Neyman-Pearson & Deming

- **Fisher**
 - Randomization & causation
 - P values
- **Neyman-Pearson**
 - Critical values: significant vs. Non-significant
 - 95% confidence intervals
- **A. E. Deming** effect sizes

<http://www.stat.ucla.edu/history/people/>



Slide 11 Statistical Summary includes elements of Fisher, Neyman-Pearson & Deming

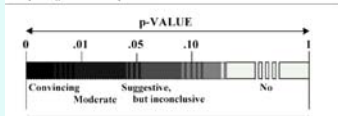
NOTES:

Anatomical abnormalities & schizophrenia

Case 2.2: 15 pairs of twins, paired t test

<http://bmj.com/cgi/content/full/322/7280/0>

Interpreting the size of a p -value

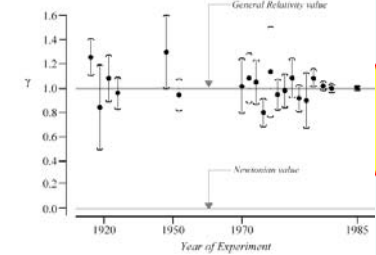


Don't use the Neyman-Pearson decision rule approach: 'significant' vs. 'Non-significant'.



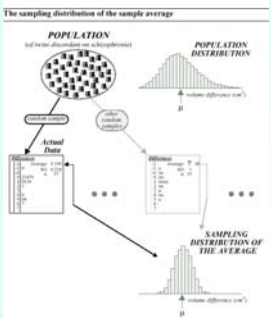
Slide 12 Confidence Intervals: Egon Pearson's major contribution

NOTES:

<p>Confidence Intervals: Egon Pearson's major contribution</p> <p>Estimates and confidence intervals for γ, the deflection of light around the sun, from 20 experiments</p>  <p>See Deb Mayo's book 'Error & the Growth of Knowledge'</p> <p>EEOS611</p>	<p>Slide 13 Confidence Intervals: Egon Pearson's major contribution</p>
	<p>NOTES:</p>

Background information on the one-sample t-tools and paired t-test

Display 2.3
The sampling distribution of the sample average



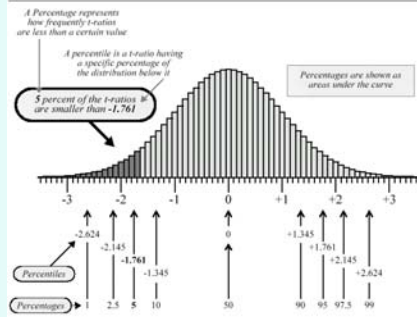
No matter what the underlying distribution, the sampling distribution of the sample averages will be 'more nearly normal' than the underlying distribution. This is a result of the **Central Limit Theorem**.

<http://mathworld.wolfram.com/CentralLimitTheorem.html>

<p>Display 2.4</p> <p>The relationship between the population distribution and the sampling distribution of the average in random sampling</p> <p>EEOS611</p>	<p>Slide 16</p> <p>NOTES:</p>
<p>Expressing uncertainty</p> <p>$X \pm \delta$, >4 choices for δ {del}</p> <ul style="list-style-type: none"> • \pm standard deviation <ul style="list-style-type: none"> • Advantage: expressed in the same units as the parameter being estimated • Not a good choice, some statisticians argue that it is inappropriate for a sample statistic • \pm standard error, also called the standard deviation of the average or sd of the mean or the standard error of the mean <ul style="list-style-type: none"> • $SE(\bar{x}) = s/\sqrt{n}$, d.f.=(n-1) • Advantage: statistically appropriate • Needs the sample size for interpretation • \pm half the 95% confidence interval <ul style="list-style-type: none"> • Assumes an underlying model for the data • For asymmetric 95% CIs in the statistics natural scale, provide the upper and lower 95% CI (for transformed data and results of Monte Carlo simulations) • Analytical precision of the instrument or technique <ul style="list-style-type: none"> • E.g., the ChI method has a certain analytical precision. This is rarely acceptable. • Hurlbert's $E(S_{100})$, a measure of species richness, has an analytical precision based on the sampling properties of the hypergeometric distribution, • Polling data has an analytical precision based on $var=p*(1-p)$, but this rarely expresses the other sources of variability in a poll 	<p>Slide 17 Expressing uncertainty</p> <p>NOTES:</p>
<p>The Z-ratio & t-ratio based on a sample average</p> <ul style="list-style-type: none"> • Z-ratio = (Estimate - Parameter)/SD(Estimate) <ul style="list-style-type: none"> • If the sampling distribution is normal, then the sampling distribution of Z is standard normal • Mean zero and standard deviation of 1 • Z distribution provided in Appendix A.1 • t-ratio = (Estimate - Parameter)/SE (Estimate) • If \bar{x} is the average in a random sample of size n from a normally distributed population, the sampling distribution of t is described by the Student's t distribution on $n-1$ degrees of freedom <p>EEOS611</p>	<p>Slide 18 The Z-ratio & t-ratio based on a sample average</p> <p>NOTES:</p>

Display 2.5

Student's t-distribution on 14 degrees of freedom



Slide 19

NOTES:

Degrees of freedom

Box 1.2

Statistical tests of significance often call upon the concept of degrees of freedom. A formal definition is the following: "The degrees of freedom of a model for expected values of random variables is the excess of the number of variables [observations] over the number of parameters in the model" (Kotz & Johnson, 1982).

In practical terms, the number of degrees of freedom associated with a statistic is equal to the number of its independent components, i.e. the total number of components used in the calculation minus the number of parameters one had to estimate from the data before computing the statistic. For example, the number of degrees of freedom associated with a variance is the number of observations minus one (noted $v = n - 1$): n components $(x_i - \bar{x})$ are used in the calculation, but one degree of freedom is lost because the mean of the statistical population is estimated from the sample data; this is a prerequisite before estimating the variance.

There is a different t distribution for each number of degrees of freedom. The same is true for the F and χ^2 families of distributions, for example. So, the number of degrees of freedom determines which statistical distribution, in these families (t , F , or χ^2), should be used as the reference for a given test of significance. Degrees of freedom are discussed again in Chapter 6 with respect to the analysis of contingency tables.

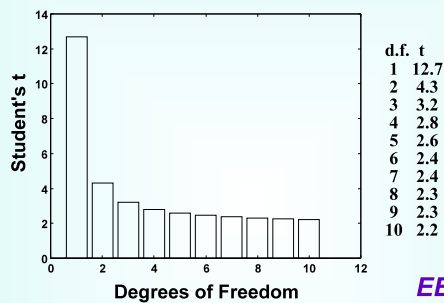
Legendre & Legendre (1989) Numerical Ecology 2nd Ed.

Slide 20

NOTES:

Student's t and sample size

3 replicates, 95% CI is ± 4.3 standard errors



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Slide 21 Student's t and sample size

NOTES:

95% CI for the mean difference

Display 2.6 (page 27) Schizophrenia study

1. Compute differences.
Obtain their averages, \bar{Y} , and standard deviation, s .
 Y_i is the volume for the unaffected twin minus the volume for the schizophrenic twin ($i = 1, \dots, 15$).
Sample average: $\bar{Y} = 199 \text{ cm}^3$.
Sample standard deviation: $s = 238 \text{ (cm}^3\text{)}, 14 \text{ d.f.}$
2. Compute $SE(\bar{Y}) = s/\sqrt{n}$ and $d.f. = n - 1$.
 $SE(\bar{Y}) = 238/\sqrt{15} = 60.15 \text{ (cm}^3\text{)}$
3. Paired t -test for the hypotheses that the population mean difference is zero.
Compute the t -statistic for this hypothesis ($T = (Y - \mu_0)/SE(\bar{Y})$).
 $t\text{-statistic} = 199/60.15 = 3.236$.
Find the p -value (two-sided here) as the proportion of values in a t_{n-1} distribution as far or further from zero than the observed t -statistic.
two-sided p -value = .006 (from computer or Table A.2 and interpolation)
4. 95% confidence interval for population mean difference.
Find the 97.5th percentile from the t -distribution or $n - 1$ degrees of freedom.
 $t_{.975} = 2.145$ (from Table A.2).
95% confidence intervals:
 $\bar{Y} \pm t_{.975} \times SE(\bar{Y})$
 $199 \text{ cm}^3 \pm 2.145 \times 60.15 \text{ cm}^3$
 $0.067 \text{ cm}^3 \text{ to } 0.331 \text{ cm}^3$

Slide 22 95% CI for the mean difference

NOTES:

A t -ratio for two-sample inference

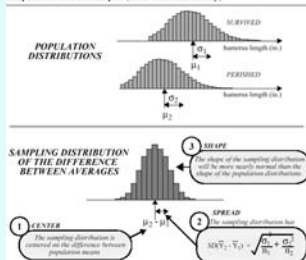
Slide 23 A t -ratio for two-sample inference

NOTES:

Sampling distribution of the difference of averages

Display 2.7 (38): Application of the Central Limit Theorem

Facts about the sampling distribution of the difference of averages from two independent random samples (from statistical theory)



Slide 24 Sampling distribution of the difference of averages

NOTES:

Pooled standard deviation

& standard error for the difference
This estimate assumes equal variances (Sleuth p 39)

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1 + n_2 - 2)}}, \quad \text{d.f.} = n_1 + n_2 - 2.$$

$$SE(\bar{Y}_2 - \bar{Y}_1) = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}.$$

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Slide 25 Pooled standard deviation

NOTES:

SE of difference

Calculation of the pooled estimate of SD and the standard error for the difference between two sample averages; Bumpus' data

1 SUMMARY STATISTICS			
Group	n	Average (in.)	Sample SD (in.)
1: Died	24	.72702	.02354
2: Survived	35	.73800	.01984

2 THE POOLED SD			
s_p	=	$\sqrt{\frac{(24-1)(.02354)^2 + (35-1)(.01984)^2}{(24 + 35 - 2)}}$	
	=	$\sqrt{\frac{.026128}{57}}$	These are the degrees of freedom associated with the pooled SD
	=	$\sqrt{.0004584}$	This is the pooled variance
Answer = s_p	=	0.02141 inches	

3 THE STANDARD ERROR			
$SE(\bar{Y}_2 - \bar{Y}_1)$	=	$0.02141 \sqrt{\frac{1}{24} + \frac{1}{35}}$	
	=	0.00567 inches	Answer

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Slide 26 SE of difference

NOTES:

95% Confidence Limits

For the difference between means

100(1- α)% Confidence Limits for the Difference Between Means

$$(\bar{Y}_2 - \bar{Y}_1) \pm t_{df}(1-\alpha/2)SE(\bar{Y}_2 - \bar{Y}_1).$$

"A 95% confidence interval will contain the parameter if the t-ratio from the observed data happens to be one of those in the middle 95% of the sampling distribution. Since 95% of all possible pairs of samples lead to such t-ratios, it is safe to say that the procedure of constructing a 95% CI is successful in 95% of its applications."

It is incorrect to say that there is a 95% probability that the true parameter is within the 95% CI. That probability is either 0 or 1. Bayesians have a different interpretation of p values.

Slide 27 95% Confidence Limits

NOTES:

CI for difference of means

Sleuth 2e Display 2.9 (41)

Construction of a 95% confidence interval for the difference between the mean humerus lengths of sparrows that died and that survived

Group	n	Average (in.)	SD (in.)
1: Died	24	.72792	.02354
2: Survived	35	.73800	.01984

$\bar{Y}_2 - \bar{Y}_1 = .73800 - .72792 = 0.01008$
 $SE(\bar{Y}_2 - \bar{Y}_1) = 0.00567$ inches
degrees of freedom = $24 + 35 - 2 = 57$
 $t_{57}(.975) = 2.002$
Half-width = $(2.002)(0.00567) = 0.01136$
Lower 95% confidence limit = $0.01008 - 0.01136 = -0.00128$ inches
Upper 95% confidence limit = $0.01008 + 0.01136 = 0.02144$ inches

From Display
2.8

from tables of the
t-distribution with
57 degrees of freedom

Slide 28 CI for difference of means

NOTES:

Testing a hypotheses about the difference of means

$$t\text{-statistic} = \frac{(\bar{Y}_2 - \bar{Y}_1) - [\text{Hypothesized value for } (\mu_2 - \mu_1)]}{SE(\bar{Y}_2 - \bar{Y}_1)}$$

"The p value for a t test is a probability of obtaining a t ratio as extreme or more extreme than the t statistic in its evidence against the null hypothesis, if the null hypothesis is correct" (Sleuth 2nd ed p 42) [Bayesians do not use this interpretation]
A large p value means that the study is not capable of excluding the null hypothesis as a possible explanation. **It is wrong to conclude that the null hypothesis is true**

Slide 29 Testing a hypotheses about the difference of means

NOTES:

Display 2.10

Was the difference consistent with chance?

The t-test for the hypothesis that the mean humerus lengths of sparrows that died is the same as the mean for sparrows that survived

Group	n	Average (in.)	SD (in.)
1: Died	24	.72792	.02354
2: Survived	35	.73800	.01984

$\bar{Y}_2 - \bar{Y}_1 = .73800 - .72792 = 0.01008$
 $SE(\bar{Y}_2 - \bar{Y}_1) = 0.00567$ inches
degrees of freedom = $24 + 35 - 2 = 57$

t-statistic = $\frac{0.01008 - 0.0}{0.00567} = 1.778$

P = .960

1-sided p-value = .040 or 2-sided p-value = $2(.040) = .080$

From Display
2.8

Hypothesized Difference

from tables of the t-distribution
with 57 degrees of freedom:
 $1.778 = t_{57}(.960)$

Slide 30 Display 2.10

NOTES:

<div data-bbox="319 170 704 205" data-label="Section-Header"> <h3>Randomization distribution</h3> </div> <div data-bbox="328 212 678 258" data-label="Text"> <p>Can be done with Matlab & R, not SPSS Display 2.11, page 46</p> </div> <div data-bbox="230 254 477 487" data-label="List-Group"> <ul style="list-style-type: none"> • Creativity data <ul style="list-style-type: none"> ▸ Randomly shuffle (500 times) the membership in intrinsic and extrinsic groups ▸ Calculate the t-ratio for each random shuffle ▸ Order the value of the t ratios from smallest to largest ▸ For a 1-sided test, calculate how many the t ratios were larger (or smaller) than the observed t ratio, add 1, and divide the number of randomizations ▸ For a 2-sided test, find the number of ratios whose absolute value exceed observed t ratios, add 1, and divide number of randomizations </div> <div data-bbox="477 291 776 541" data-label="Figure"> </div>	<div data-bbox="816 134 1330 170" data-label="Section-Header"> <h3>Slide 31 Randomization distribution</h3> </div> <div data-bbox="816 258 940 291" data-label="Text"> <p>NOTES:</p> </div>
<div data-bbox="271 653 721 718" data-label="Section-Header"> <h3>Randomization doesn't solve problems with unequal variance</h3> </div> <div data-bbox="256 741 534 1001" data-label="List-Group"> <ul style="list-style-type: none"> • Randomization is often superior to the t-distribution for 2-sample problems • Randomization does not remedy violations of the assumptions of the t test. <ul style="list-style-type: none"> ▸ The most common problem with Student's t test is the so-called Fisher-Behrens problem, testing the difference in the average if the distributions have different variances ▸ This is an open question ▸ Neither nonparametric approaches (see Chapter 4) nor randomization provide a clear solution </div> <div data-bbox="621 816 716 959" data-label="Image"> </div> <div data-bbox="654 997 779 1024" data-label="Text"> <p>EEOS611</p> </div>	<div data-bbox="816 623 1343 697" data-label="Section-Header"> <h3>Slide 32 Randomization doesn't solve problems with unequal variance</h3> </div> <div data-bbox="816 783 940 816" data-label="Text"> <p>NOTES:</p> </div>
<div data-bbox="289 1253 742 1329" data-label="Section-Header"> <h3>Chapter 3: A closer look at assumptions</h3> </div>	<div data-bbox="816 1148 1344 1222" data-label="Section-Header"> <h3>Slide 33 Randomization doesn't solve problems with unequal variance</h3> </div> <div data-bbox="816 1308 940 1341" data-label="Text"> <p>NOTES:</p> </div>

Randomization doesn't solve problems with unequal variance

- Randomization is often superior to the t-distribution for 2-sample problems
- Randomization does not remedy violations of the assumptions of the t test.
 - ▶ The most common problem with Student's t test is the so-called Fisher-Behrens problem, testing the difference in the average if the distributions have different variances
 - ▶ This is an open question
 - ▶ Neither nonparametric approaches (see Chapter 4) nor randomization provide a clear solution



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Slide 34 Chapter 3: A closer look at assumptions

NOTES:

Case study 3.1

Cloud seeding to increase rainfall — A randomized experiment

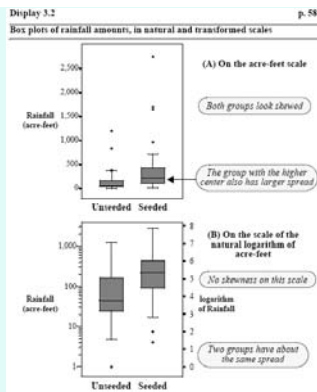
- 52-day experiment
- Random selection each day to seed or not to seed a cloud; pilot 'blind' to treatment
- Rainfall measured
- Data highly skewed

display 3.1

Rainfall (acre-feet) for days with and without cloud seeding											
Rainfall from unseeded days (n = 26)											
1202.6	830.1	372.4	345.5	321.2	244.3	163.0	147.8	95.0			
87.0	81.2	68.5	47.3	41.1	36.6	29.0	28.6	26.3			
26.1	24.4	21.7	17.3	11.5	4.9	4.9	1.0				
Rainfall from seeded days (n = 26)											
2745.6	1697.8	1656.0	978.0	703.4	489.1	430.0	334.1	302.8			
274.7	274.7	255.0	242.5	200.7	198.6	129.6	119.0	118.3			
115.3	92.4	66.6	32.7	31.4	17.5	7.7	4.1				

Slide 35 Case study 3.1

NOTES:



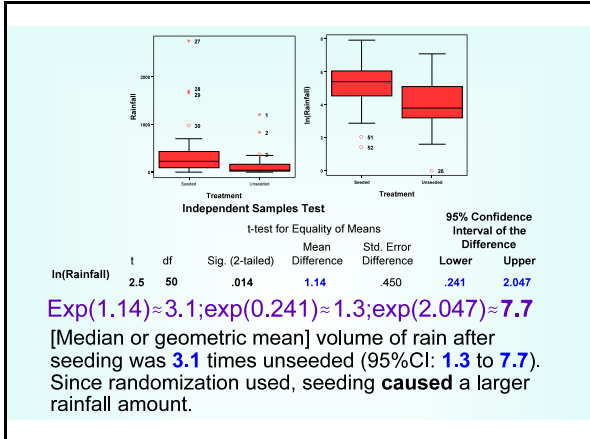
Statistical Inference
 [Median or geometric mean] volume of rain after seeding was 3.1 times unseeded (95%CI: 1.3 to 7.7). Since randomization used, seeding **caused** a larger rainfall amount.

Slide 36

NOTES:

Slide 37

NOTES:

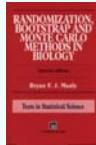


Randomization doesn't solve problems with unequal variance

- Randomization is often superior to the nominal p values from the t-distribution for 2-sample problems, but randomization does not remedy the most important problem with the t distribution: unequal variance

- The most common problem with Student's t test is the so-called Fisher-Behrens problem, testing the difference between averages from distributions with different variances

- This is an open question
- Neither nonparametric approaches (see Chapter 4) nor randomization provide a clear solution



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Slide 38 Randomization doesn't solve problems with unequal variance

NOTES:

Case 3.2: Dioxin study

Differences between veteran dioxin concentrations could be due to chance

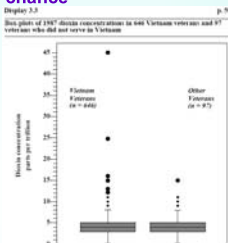
- 646 Veterans who served in Viet Nam during 1967 & 1968 in areas treated with Agent Orange

- 97 other veterans served between 1965-1971 in US or Germany

- Serum dioxin levels measured

- Statistical Summary:

- No evidence that the mean dioxin levels differ (1-sided p value=0.4)
- Extrapolation speculative; dioxin-affected vets may not have participated in the survey



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Slide 39 Case 3.2: Dioxin study

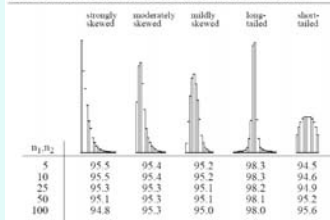
NOTES:

<div data-bbox="315 237 711 315" data-label="Section-Header"> <h2>Robustness of the two-sample t tools</h2> </div>	<div data-bbox="815 132 1375 205" data-label="Section-Header"> <h3>Slide 40 Robustness of the two-sample t tools</h3> </div> <div data-bbox="815 294 940 325" data-label="Text"> <p>NOTES:</p> </div>
<div data-bbox="354 693 662 730" data-label="Section-Header"> <h2>Assumptions of t test</h2> </div> <div data-bbox="250 766 734 1041" data-label="List-Group"> <ul style="list-style-type: none"> Two major assumptions <ul style="list-style-type: none"> Both samples are independent samples from normally distributed populations Both samples have identical standard deviations The t tests are usually robust to modest violations of the assumptions <ul style="list-style-type: none"> These assumptions are never strictly met, but the t test is remarkably robust to violations of the assumptions Robust means the conclusions from test — e.g., p values, confidence limits — are valid even when the assumptions aren't strictly met, especially if sample sizes nearly equal Transformations of the data are often used </div> <div data-bbox="652 1033 779 1062" data-label="Text"> <p>EEOS611</p> </div>	<div data-bbox="815 659 1239 697" data-label="Section-Header"> <h3>Slide 41 Assumptions of t test</h3> </div> <div data-bbox="815 781 940 814" data-label="Text"> <p>NOTES:</p> </div>
<div data-bbox="298 1176 732 1239" data-label="Section-Header"> <h2>Violations of assumptions that matter</h2> </div> <div data-bbox="245 1260 725 1499" data-label="List-Group"> <ul style="list-style-type: none"> With equal sample sizes, the t-test is affected moderately by long-tailedness (leptokurtic or peaked distribution) and very little by skewness (the symmetry of the distribution) <ul style="list-style-type: none"> Kurtosis: peakedness, platykurtic (flat distribution), leptokurtic (peaked) Skewness: symmetry If the two populations have the same standard deviations and approximately the same shape, with unequal sample size, the t tests are affected moderately by long tailedness (leptokurtic) and substantially by skewness If the skewness differs considerably, the tools can be misleading with small and moderate sample sizes </div> <div data-bbox="652 1520 779 1549" data-label="Text"> <p>EEOS611</p> </div>	<div data-bbox="815 1146 1367 1218" data-label="Section-Header"> <h3>Slide 42 Violations of assumptions that matter</h3> </div> <div data-bbox="815 1306 940 1339" data-label="Text"> <p>NOTES:</p> </div>

Monte Carlo simulations of violations on p values

Display 3.4

Percentage of 95% confidence intervals that are successful when the two populations are non-normal (but same shape and SD, and equal sample sizes); each percentage is based on 1,000 computer simulations



really matter?

When 2 populations are the same shape with equal n, the t test is only affected moderately (conservative for leptokurtic [peaked] distributions)

Slide 43 Monte Carlo simulations of violations on p values

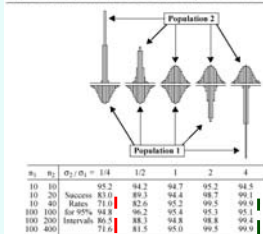
NOTES:

Different standard deviations & sample sizes

Robust if sample sizes the same, nonconservative if smaller group has higher s.d.

Display 3.5

Percentage of successful 95% confidence intervals when the two populations have different standard deviations (but are normal) with possibly different sample sizes; each percentage is based on 1,000 computer simulations



P (Type I error) >> stated value (e.g., 0.05) if sd of smaller group larger than larger group

P (Type I error) << stated value (e.g., 0.05) if sd of smaller group smaller than larger group

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Slide 44 Different standard deviations & sample sizes

NOTES:

Departures from independence

Cluster, serial & spatial effects can be serious and are more difficult to account for in statistical analysis

- Cluster effects
 - Mice from litters
 - Copepods from net hauls
 - Technician-to-technician variability in sample analysis
- Serial effects (an explicit time or space term)
 - Temporal autocorrelation
- Spatial effects: positive autocorrelation (Ellen Douglas's research on flood frequency, Chen & Ferguson on MCAS scores)

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- Check residuals (observed-expected) for spatial or temporal pattern
- Inferences based on Student's t tests can be very misleading or wrong if there are spatial or temporal correlations in residuals

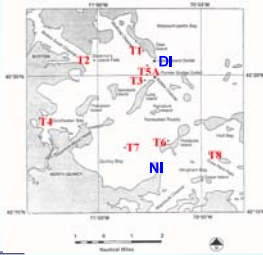
Slide 45 Departures from independence

NOTES:

MWRA Benthic Sampling Stations

8 Stations, May & Aug
3 replicate 0.043-m² Ted Young grabs; 300- μ m sieves

- T1: Deer Island
- T2: Governor's Island Flats
- T3: Long Island
- T4: Savin Hill Cove
- T5A: Presidents Road
- T6: Peddocks Island
- T7: Quincy Bay
- T8: Hingham/Hull Bay
- NI: Nut Island
- DI: Deer Island



Slide 46 MWRA Benthic Sampling Stations

NOTES:

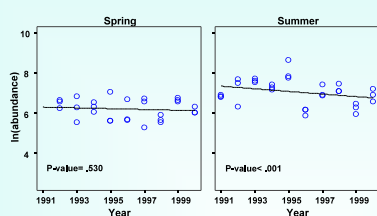


Fig. 40. The change in In(abundance) at Quincy Bay(T07). There is a significant lack of fit in both the spring and summer data to linear regression. The p-values reported are from the One-way ANOVA test.

Banik 2003 UMB M.Sc.
Problems with serial autocorrelation (confounded with spatial effects) create a problem called 'lack of fit' in OLS and regression
Solution: ANOVA test for linear trends

The residuals after fitting the regression should be identically independently normally distributed, but they are not.
The analyst can not ignore these effects and perform a regression as if these residuals are independent

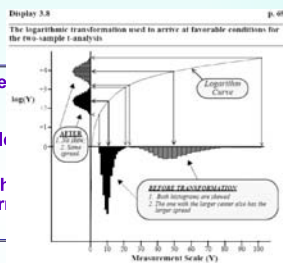
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NOTES:

Log transform of rainfall

See Case 3.1 movie

- The antilogarithm of the mean of the log values, the geometric mean, is the median on the original scale of measurement
- Calculate the 95% CI's on the log scale and back transform they will be asymmetric



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Slide 48 Log transform of rainfall

NOTES:

3.5.3 Transformations

- Log (x+1) transform
 - Most biological data, but not usually diversity
 - Needed when there is a multiplicative process in action: growth, bank account interest
 - Marine pollutants: polynuclear aromatic hydrocarbons, fecal coliform bacteria, but not usually metals
 - Calculate the mean and 95% CI and then back-transform. For symmetric data, the mean of the log-transformed data = median. Label as the geometric mean
- Many other transforms
 - Arcsin (\sqrt{Y}) for frequency data ranging between 0 and 1 (but the logit transform may be better)
 - % silt clay, but the data must be on the interval 0 to 1
 - Logit transform: $\log [Y/(1-Y)]$
 - Square roots for counts, reciprocal for waiting times, logit transforms for proportions between 0 and 1 ($\log (P/(1-P))$)
 - "... it is recommended here that a trial-and error approach, with graphical analysis, be used instead."

Slide 49 3.5.3 Transformations

NOTES:

Display 3.8 p. 68
Two-sample t-analysis and statement of conclusions after logarithmic transformation — cloud seeding example

1 Transform the data	
UNSEEDED	SEEDED
Yearly Rainfall (mm)	Yearly Rainfall (mm)
1202.0	7092
810.1	14722
372.0	9320
342.5	1841
321.2	5772
244.3	4098
163.0	5094
147.0	4094
95.0	4554
87.0	4064
81.2	4397
48.5	2227
47.3	5857
41.1	5714
36.0	1600
29.0	3367
28.0	3351
26.1	5270
26.1	5270
24.4	1191
21.7	1077
17.3	2481
13.5	2440
4.9	1589
1.0	0.000

Conclusion: There is convincing evidence that seeding increased rainfall (1-sided p-value = .0070). It is estimated that the volume of rainfall produced by a seeded cloud was 3.14 times as large as the volume that would have been produced in the absence of seeding. (95% confidence: 1.27 to 7.74 times).

Do the test and calculate the 95% CI on transformed data and then back transform the effect size and confidence limits. Report as ratio of geometric means (Sleuth: ratio of medians).

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NOTES:

<http://www.cdc.gov/exposurereport/>

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NOTES:

Table 16. Lead in blood.

Geometric mean and selected percentiles of blood concentrations (in $\mu\text{g/dL}$) for the U.S. population aged 1 year and older, National Health and Nutrition Examination Survey, 1999-2002.

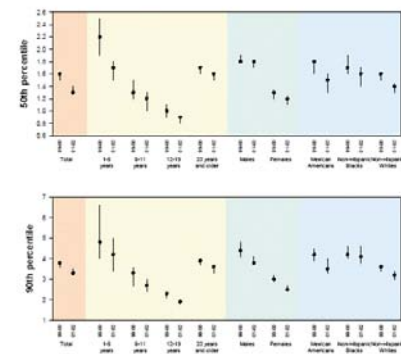
	Survey years	Geometric mean (95% conf. interval)	Selected percentiles (95% confidence interval)				Sample size
			50th	75th	90th	95th	
Total, age 1 and older	99-02	1.68 (1.60-1.72)	1.69 (1.61-1.82)	2.48 (2.30-2.62)	3.89 (3.65-4.02)	4.89 (4.60-5.20)	7910
	01-02	1.48 (1.39-1.57)	1.49 (1.30-1.45)	2.29 (2.10-2.35)	3.49 (3.10-3.85)	4.49 (4.10-4.79)	8045
Age group							
1-5 years	99-02	2.23 (1.90-2.62)	2.23 (1.90-2.62)	3.30 (2.90-3.80)	4.89 (4.10-6.00)	7.00 (5.10-9.50)	723
	01-02	1.79 (1.50-1.87)	1.59 (1.40-1.72)	2.59 (2.30-2.82)	4.19 (3.40-5.00)	5.89 (4.70-6.80)	846
5-11 years	99-02	1.51 (1.30-1.83)	1.34 (1.20-1.50)	2.09 (1.70-2.40)	3.39 (2.70-3.80)	4.59 (3.60-5.20)	805
	01-02	1.25 (1.14-1.36)	1.13 (1.00-1.30)	1.69 (1.40-1.80)	2.79 (2.40-3.00)	3.79 (3.10-4.70)	1044
12-19 years	99-02	1.19 (1.04-1.17)	1.09 (1.00-1.12)	1.49 (1.30-1.62)	2.39 (2.10-2.30)	2.89 (2.40-3.00)	2135
	01-02	0.82 (1.09-1.08)	0.99 (1.00-1.02)	1.29 (1.20-1.30)	1.99 (1.90-2.02)	2.79 (2.30-2.80)	2291
20 years and older	99-02	1.78 (1.68-1.91)	1.79 (1.65-1.70)	2.59 (2.30-2.60)	3.99 (3.70-4.00)	5.29 (4.60-6.00)	4207
	01-02	1.56 (1.49-1.62)	1.69 (1.55-1.62)	2.29 (2.20-2.30)	3.69 (3.30-3.70)	4.69 (4.20-4.80)	4772
Gender							
Males	99-02	2.01 (1.92-2.07)	1.93 (1.80-1.90)	2.59 (2.30-2.60)	4.49 (4.10-4.80)	6.00 (5.10-6.40)	2913
	01-02	1.78 (1.77-1.84)	1.79 (1.70-1.80)	2.79 (2.50-2.82)	3.99 (3.70-4.10)	5.29 (4.60-5.50)	4338
Females	99-02	1.37 (1.32-1.43)	1.39 (1.20-1.30)	1.99 (1.90-2.02)	3.09 (2.90-3.20)	4.09 (3.70-4.20)	4087
	01-02	1.19 (1.14-1.25)	1.13 (1.10-1.20)	1.69 (1.70-1.68)	2.69 (2.40-2.70)	3.69 (3.10-3.70)	4936
Race/ethnicity							
Mexican Americans	99-02	1.83 (1.75-1.91)	1.83 (1.65-1.80)	2.79 (2.50-2.80)	4.29 (3.90-4.60)	5.89 (5.10-6.60)	2742
	01-02	1.46 (1.36-1.63)	1.50 (1.30-1.60)	2.29 (2.00-2.40)	3.69 (3.30-4.00)	4.69 (4.10-5.00)	2399
Non-Hispanic blacks	99-02	1.87 (1.75-2.02)	1.79 (1.60-1.80)	2.69 (2.30-2.90)	4.29 (3.80-4.60)	5.79 (5.20-6.10)	1942
	01-02	1.69 (1.62-1.82)	1.63 (1.40-1.70)	2.39 (2.10-2.40)	4.29 (3.80-4.60)	5.79 (5.20-6.10)	2219
Non-Hispanic whites	99-02	1.62 (1.55-1.69)	1.69 (1.60-1.60)	2.49 (2.30-2.40)	3.69 (3.30-3.70)	5.09 (4.60-5.70)	2719
	01-02	1.43 (1.37-1.48)	1.49 (1.30-1.40)	2.19 (2.10-2.20)	3.19 (3.00-3.40)	4.19 (3.70-4.50)	3059

<http://www.cdc.gov/exposurereport/3rd/pdf/thirdreport.pdf>

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NOTES:

Figure 6. Lead in blood.

Selected percentiles with 95% confidence intervals of blood concentrations (in $\mu\text{g/dL}$) for the U.S. population aged 1 year and older, National Health and Nutrition Examination Survey, 1999-2002.

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NOTES:

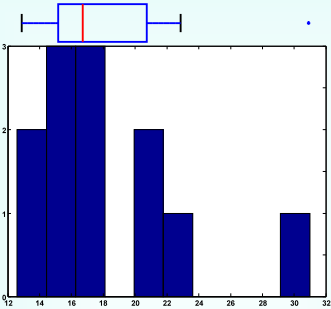
Outliers and resistant procedures

- A procedure is resistant if it doesn't change much when a small part of the data changes, perhaps drastically.
- t tools are based on averages and are strongly affected by outliers
 - Chapter 4 introduces tests based on ranks, which protect against outliers (but not against unequal variance)
- Practical strategies
 - Do side-by-side box plots to analyze departures from assumptions
 - Check for patterns in residuals with box plots
 - Consider & test for serial spatial and cluster effects
 - Analyze spatial patterns in the residuals, use more sophisticated tools
 - Legendre & Legendre: If pos. Spatial autocorrelation, decrease the p value. Test for differences at the 0.001 level instead of the 0.05 level

Slide 54 Outliers and resistant procedures

NOTES:

<p>Display 3.6 p. 66</p> <p>Examination Strategy</p>	<p>Slide 55</p> <p>NOTES:</p>
<p>Display 3.7 p. 67</p> <p>Outlier analysis for Agent Orange Data; effect of outliers on the p-value for equal population means</p> <p>Report results, with and without outliers</p>	<p>Slide 56 Identifying outliers with boxplots</p> <p>NOTES:</p>
<p>Practical strategies for outliers</p> <p>Be wary of outlier deletion!</p> <ul style="list-style-type: none"> ● Outlier strategy <ul style="list-style-type: none"> ▶ Run analysis with and without outliers ▶ Throw-out outliers only if there is very compelling evidence to do so, and document this data paring or culling ● Note that outlier removal has created tremendous problems: <ul style="list-style-type: none"> ▶ POC flux to the deep sea ▶ The ozone hole ▶ Mendel's data: 1:2:1 ratios and the chi-square test; documented by Fisher ▶ Milliken's study of the charge of the electron 	<p>Slide 57 Practical strategies for outliers</p> <p>NOTES:</p>

<div><p>Is the datum at 30 an outlier?</p><p>EEOS611</p></div>	Slide 58 Is the datum at 30 an outlier?
	NOTES: