

1. (1 pt) The solution of a certain differential equation is of the form

$$y(t) = ae^{4t} + be^{7t},$$

where  $a$  and  $b$  are constants.

The solution has initial conditions  $y(0) = 5$  and  $y'(0) = 2$ .

Find the solution by using the initial conditions to get linear equations for  $a$  and  $b$ .

$$y(t) = \underline{\hspace{2cm}}$$

2. (1 pt) It is easy to check that for any value of  $c$ , the function

$$y = ce^{-2x} + e^{-x}$$

is solution of equation

$$y' + 2y = e^{-x}.$$

Find the value of  $c$  for which the solution satisfies the initial condition  $y(-3) = 6$ .

$$c = \underline{\hspace{2cm}}$$

3. (1 pt) Find the value of  $k$  for which the constant function  $x(t) = k$  is a solution of the differential equation  $2t^5 \frac{dx}{dt} - 8x - 8 = 0$ .

4. (1 pt) Match each of the following differential equations with a solution from the list below.

- \_\_\_1.  $y'' - 9y' + 20y = 0$
- \_\_\_2.  $2x^2y'' + 3xy' = y$
- \_\_\_3.  $y'' + y = 0$
- \_\_\_4.  $y'' + 9y' + 20y = 0$

A.  $y = e^{-5x}$

B.  $y = \frac{1}{x}$

C.  $y = e^{4x}$

D.  $y = \cos(x)$

5. (1 pt) It is easy to check that for any value of  $c$ , the function

$$y = x^2 + \frac{c}{x^2}$$

is solution of equation

$$xy' + 2y = 4x^2, \quad (x > 0).$$

Find the value of  $c$  for which the solution satisfies the initial condition  $y(3) = 2$ .

$$c = \underline{\hspace{2cm}}$$

6. (1 pt)

Newton's law of cooling says that the rate of cooling of an object is proportional to the difference between the temperature of the object and that of its surroundings (provided the difference is not too large).

If  $T = T(t)$  represents the temperature of a (warm) object at time  $t$ ,  $A$  represents the ambient (cool) temperature, and  $k$  is a negative constant of proportionality, which equation(s) accurately characterize Newton's law?

- A.  $\frac{dT}{dt} = kT(1 - T/A)$
- B.  $\frac{dT}{dt} = k(A - T)$
- C.  $\frac{dT}{dt} = kT(T - A)$
- D.  $\frac{dT}{dt} = k(T - A)$
- E. All of the above
- F. None of the above