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1. (1 pt) Find a function y of x such that

$$9yy' = x \text{ and } y(9) = 10.$$

$$y = \underline{\hspace{2cm}}.$$

2. (1 pt) Solve the separable differential equation for u

$$\frac{du}{dt} = e^{3u+2t}.$$

Use the following initial condition: $u(0) = 6$.

$$u = \underline{\hspace{2cm}}.$$

3. (1 pt)

A country has 10 billion dollars in paper currency in circulation, and each day 30 million dollars comes into the country's banks. The government decides to introduce new currency by having the banks replace old bills with new ones whenever old currency comes into the bank. Let $x = x(t)$ denote the number of new dollars in circulation after t days with units in billions and $x(0) = 0$.

A. Determine a differential equation which describes the rate at which x is growing:

$$\frac{dx}{dt} = \underline{\hspace{2cm}}$$

B. Solve the differential equation subject to the initial conditions given above.

$$x(t) = \underline{\hspace{2cm}}$$

C. How many days will it take for the new bills to account for 90 percent of the currency in circulation? $\underline{\hspace{2cm}}$

HINT: The 30 million dollars coming in daily include both old and new currency, in amounts proportional to the percentage of each in circulation at the time.

4. (1 pt)

A tank is filled with 1000 liters of pure water. Brine containing 0.04 kg of salt per liter enters the tank at 9 liters per minute. Another brine solution containing 0.06 kg of salt per liter enters the tank at 6 liters per minute. The content of the tank is

kept thoroughly mixed and drains from the tank at 15 liters per minute.

A. Determine the differential equation which describes this system. Let $S(t)$ denote the amount of salt (in kg) in the tank after t minutes. Then

$$\frac{dS}{dt} = \underline{\hspace{2cm}}$$

B. Solve the differential equation for $S(t)$.

$$S(t) = \underline{\hspace{2cm}}$$

5. (1 pt)

A cup of coffee at 178 degrees is poured into a mug and left in a room at 74 degrees. After 7 minutes, the coffee is 146 degrees. Assume that the differential equation describing Newton's Law of Cooling is (in this case) $\frac{dT}{dt} = k(T - 74)$.

What is the temperature of the coffee after 17 minutes?

After how many minutes will the coffee be 100 degrees?

6. (1 pt)

A bacteria culture starts with 800 bacteria and grows at a rate proportional to its size. After 6 hours, there are 10000 bacteria.

A. Find an expression for the number of bacteria after t hours.

B. Find the number of bacteria after 7 hours.

C. Find the growth rate after 7 hours.

D. After how many hours will the population reach 30000?
