

1. (1 pt) Find all the values of  $x$  such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(5x)^n}{n^4}$$

The series is convergent

from  $x = \underline{\hspace{1cm}}$ , left end included (enter Y or N):  $\underline{\hspace{1cm}}$

to  $x = \underline{\hspace{1cm}}$ , right end included (enter Y or N):  $\underline{\hspace{1cm}}$

2. (1 pt) Find all the values of  $x$  such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$$

Answer:  $\underline{\hspace{2cm}}$

**Note:** Give your answer in **interval notation**

3. (1 pt) Find all the values of  $x$  such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(5x-9)^n}{n^2}$$

Answer:  $\underline{\hspace{2cm}}$

**Note:** Give your answer in **interval notation**

4. (1 pt) The function  $f(x) = \ln(1-x^2)$  is represented as a power series

$$f(x) = \sum_{n=0}^{\infty} c_n x^n.$$

Find the FOLLOWING coefficients in the power series.

$$c_0 = \underline{\hspace{1cm}}$$

$$c_1 = \underline{\hspace{1cm}}$$

$$c_2 = \underline{\hspace{1cm}}$$

$$c_3 = \underline{\hspace{1cm}}$$

$$c_4 = \underline{\hspace{1cm}}$$

Find the radius of convergence  $R$  of the series.

$$R = \underline{\hspace{1cm}}.$$

5. (1 pt) Suppose that

$$\frac{5x}{x+11} = \sum_{n=0}^{\infty} c_n x^n.$$

Find the following coefficients.

$$c_0 = \underline{\hspace{1cm}}$$

$$c_1 = \underline{\hspace{1cm}}$$

$$c_2 = \underline{\hspace{1cm}}$$

$$c_3 = \underline{\hspace{1cm}}$$

$$c_4 = \underline{\hspace{1cm}}$$

Find the radius of convergence  $R$  of the power series.

$$R = \underline{\hspace{1cm}}$$

6. (1 pt) (a)

Evaluate the integral

$$\int_0^2 \frac{24}{x^2+4} dx.$$

Your answer should be in the form  $k\pi$ , where  $k$  is an integer.

What is the value of  $k$ ?

$$\text{Hint: } \frac{d \arctan(x)}{dx} = \frac{1}{x^2+1}$$

$$k = \underline{\hspace{1cm}}$$

(b)

Now, let's evaluate the same integral using power series. First, find the power series for the function  $f(x) = \frac{24}{x^2+4}$ . Then, integrate it from 0 to 2, and call it  $S$ .  $S$  should be an infinite series.

What are the first few terms of  $S$ ?

$$a_0 = \underline{\hspace{1cm}}$$

$$a_1 = \underline{\hspace{1cm}}$$

$$a_2 = \underline{\hspace{1cm}}$$

$$a_3 = \underline{\hspace{1cm}}$$

$$a_4 = \underline{\hspace{1cm}}$$

(c) The answers to part (a) and (b) are equal (why?). Hence, if you divide your infinite series from (b) by  $k$  (the answer to (a)), you have found an estimate for the value of  $\pi$  in terms of an infinite series. Approximate the value of  $\pi$  by the first 5 terms.

$$\underline{\hspace{2cm}}.$$

(d)

What is the upper bound for your error of your estimate if you use the first 11 terms? (Use the alternating series estimation.)

$$\underline{\hspace{2cm}}.$$