UMass Boston
Math 140, Summer 2007
First Exam
August 8, 2007

Name: ..... Student ID: .....

Directions:

(1) PRINT your name on the blue book and on this test on the designated spot.

(2) This test is closed-book, no-calculator (don't panic, you won't need one!)

(3) Partial credit may be given for an incomplete answer, but NO CREDIT is given for an answer that: is not relevant to the question asked, OR does not give an explanation when this is asked for, OR does not show the computational work, OR cannot be read or understood. (Be wise: keep the grader happy!)

(4) Explanations should be *succinct*: "brief and to the point."

(5) This test lasts 105 minutes and is worth 100 points, with the point distribution for each problem denoted by ( )'s throughout the test.

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(1) **(16 p)** Find the following limits and show your work.

$$a) \quad \lim_{x\to 4} \frac{x^2-4x}{x^2-3x-4} \qquad b) \quad \lim_{x\to 0} \frac{4x}{\tan(3x)}$$

$$c) \quad \lim_{x\to 3^-} \frac{x+3}{x^2-9} \qquad d) \quad \lim_{x\to 0^+} \frac{x}{\sqrt{x^2+1}-1}$$

(2) **(10 p)** Find the constant c that makes the function

$$g(x) = \begin{cases} x^2 - c, & \text{if } x < 4 \\ cx + 1, & \text{if } x \geqslant 4 \end{cases}$$

continuous on $(-\infty, \infty)$. Is there any value of c for which the function is differentiable on $(-\infty, \infty)$?

(3) **(10 p)** Show that the equation $x^4 + x - 3 = 0$ has at least one real root. Find an interval of length 0.5 that contains a root of the equation.

(4) **(14 p)** Let $f(x) = \sqrt{3x+1}$.
- (a) Use the limit definition of the derivative to find $f'(5)$. **Note that you must use the definition to get any credit.**
- (b) Use differentiation laws to find $f'(5)$.
- (c) Find the equation of the line tangent to the graph of f at the point corresponding to $x = 5$.

(5) **(18 p)** Find the required derivative(s) for each of the following functions:

$$f(x) = -9x^5 + 4x^3 - 6x^2 - 1 \qquad \text{Find } f'(x) \text{ and } f''(x).$$

$$h(x) = x^2 \cos x \qquad \text{Find } h'(x)$$

$$g(y) = \tan(2y) \qquad \text{Find } g'(y) \text{ and } g''(\pi)$$

$$t(x) = \frac{x}{x^2+1} \qquad \text{Find } t'(x) \text{ and } t'(1).$$

(6) **(10 p)** The curve given by the implicit equation $x^2+xy+y^2 = 3$ is an ellipse. Find the point(s) on the curve where the tangent line is horizontal, and the point(s) where the tangent line is vertical.

(7) **(12 p)** A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away at a rate of 2 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 8ft from the wall?

(8) **(10 p)** Let $f(x) = \frac{1}{\sqrt{x}}$.
- (a) Find the linearization of f at $a = 1$.
- (b) Use this linearization to estimate $\frac{1}{\sqrt{1.01}}$.