Curvilinear Coordinates

January 27, 2010

Polar Coordinates

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- θ: angle from Ox to OQ measured counterclockwise

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- r: $[0, \infty)$
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Range of coordinates:

- r: $[0, \infty)$
- θ:$[0, 2\pi)$, for example

Transition Polar - Rectangular

Polar to rectangular:

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Polar to rectangular:

$$x = r\cos\theta \quad , \quad y = r\sin\theta$$

Rectangular to polar:

Transition Polar - Rectangular

Polar to rectangular:

$$x = r\cos\theta \quad , \quad y = r\sin\theta$$

Rectangular to polar:

$$r = \sqrt{x^2 + y^2}$$

$$\cos\theta = \frac{x}{r} \quad , \quad \sin\theta = \frac{y}{r} \ .$$

Cylindrical coordinates

$$P(x, y, z) \Longleftrightarrow P(r, \theta, z)$$

- Polar in Oxy – (r, θ);
- Rectangular in Orz – (r, z).

Cylindrical to rectangular:

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$$x = r\cos\theta \quad , \quad y = r\sin\theta \quad , \quad z_r = z_c$$

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$$\cos\theta = \frac{x}{r} \quad , \quad \sin\theta = \frac{y}{r} \, .$$

Constant Coordinate Sets

Surfaces:

- r constant

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- r constant $\longrightarrow$ vertical cylinder;
- θ constant

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Surfaces:

- r constant $\longrightarrow$ vertical cylinder;
- θ constant $\longrightarrow$ vertical half plane;
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Surfaces:

- r constant $\longrightarrow$ vertical cylinder;
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- r constant $\longrightarrow$ vertical cylinder;
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Curves:

- θ, z constant $\longrightarrow$ horizontal ray;
- r, z constant

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Surfaces:

- r constant $\longrightarrow$ vertical cylinder;
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Curves:

- θ, z constant $\longrightarrow$ horizontal ray;
- r, z constant $\longrightarrow$ horizontal circle;
- r, θ constant

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- θ constant $\longrightarrow$ vertical half plane;
- z constant $\longrightarrow$ horizontal plane.

Curves:

- θ, z constant $\longrightarrow$ horizontal ray;
- r, z constant $\longrightarrow$ horizontal circle;
- r, θ constant $\longrightarrow$ vertical line.

Spherical Coordinates

Coordinates:

- ρ: distance $|OP|$;
- ϕ: angle Oz to OP;
- θ: angle Ox to OP_{xy}.

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$$z = \rho\cos\phi$$

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Rectangular to spherical:

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Rectangular to spherical:

$$\rho = \sqrt{x^2 + y^2 + z^2} \quad , \quad r = \sqrt{x^2 + y^2}$$
$$\cos\phi = \frac{z}{\rho} \quad , \quad \sin\phi = \frac{r}{\rho}$$
$$\cos\theta = \frac{x}{r} \quad , \quad \sin\theta = \frac{y}{r}$$

Curvilinear boxes

Curvilinear boxes

Cylindrical "box":

$$X = \{P(r, \theta, z) \mid 0 \leqslant r \leqslant r_0\,, 0 \leqslant \theta \leqslant \theta_0\,, 0 \leqslant z \leqslant z_0\}$$

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