

1. (1 pt)

Find the directional derivative of $f(x,y) = x^2y^3 + 2x^4y$ at the point $(4, 0)$ in the direction $\theta = 2\pi/3$.

The gradient of f is:

$$\nabla f = \langle \text{_____}, \text{_____} \rangle$$

$$\nabla f(4,0) = \langle \text{_____}, \text{_____} \rangle$$

The directional derivative is:

2. (1 pt)

Consider the function $f(x,y) = 5xy^2 + 2x^3y$.

Find the gradient of f :

$$\langle \text{_____}, \text{_____} \rangle$$

Find the gradient of f at the point $(1, 0)$.

$$\langle \text{_____}, \text{_____} \rangle$$

Find the rate of change of the function f at the point $(1, 0)$ in the direction $\mathbf{u} = \langle -4/13, \sqrt{153}/13 \rangle$.

3. (1 pt)

Consider the function $f(x,y,z) = xy + yz^2 + xz^3$.

Find the gradient of f :

$$\langle \text{_____}, \text{_____}, \text{_____} \rangle$$

Find the gradient of f at the point $(-3, 1, 5)$.

$$\langle \text{_____}, \text{_____}, \text{_____} \rangle$$

Find the rate of change of the function f at the point $(-3, 1, 5)$ in the direction $\mathbf{u} = \langle 5/\sqrt{66}, 5/\sqrt{66}, -4/\sqrt{66} \rangle$.

4. (1 pt)

Find the directional derivative of $f(x,y,z) = z^3 - x^2y$ at the point $(-3, 3, -3)$ in the direction of the vector $\mathbf{v} = \langle -1, 1, -1 \rangle$.

5. (1 pt)

The temperature at a point (x,y,z) is given by $T(x,y,z) = 200e^{-x^2-y^2/4-z^2/9}$, where T is measured in degrees celcius and x,y , and z in meters. There are lots of places to make silly errors

in this problem; just try to keep track of what needs to be a unit vector.

Find the rate of change of the temperature at the point $(1, -1, 2)$ in the direction toward the point $(0, 3, -4)$.

In which direction (unit vector) does the temperature increase the fastest at $(1, -1, 2)$?

$$\langle \text{_____}, \text{_____}, \text{_____} \rangle$$

What is the maximum rate of increase of T at $(1, -1, 2)$?

6. (1 pt)

Suppose that you are climbing a hill whose shape is given by $z = 498 - 0.09x^2 - 0.07y^2$, and that you are at the point $(25, 45, 300)$.

In which direction (unit vector) should you proceed initially in order to reach the top of the hill fastest?

$$\langle \text{_____}, \text{_____}, \text{_____} \rangle$$

If you climb in that direction, at what angle above the horizontal will you be climbing initially (radian measure)?

$$\text{_____}$$

7. (1 pt)

Find equations of the tangent plane and normal line to the surface $z - 2 = xe^y \cos z$ at the point $(-2, 0, 0)$.

Tangent Plane: (make the coefficient of z equal to 1).

$$\text{_____} = 0.$$

$$\text{Normal line: } \langle -2, \text{_____}, \text{_____} \rangle$$

$$+t \langle \text{_____}, \text{_____}, 1 \rangle.$$

8. (1 pt)

Find the points on the surface $5x^2 + 3y^2 + 5z^2 = 1$ at which the tangent plane is parallel to the plane $4x + 2y + 3z = 7$.

$$(\text{_____}, \text{_____}, \text{_____}) \text{ and}$$

$$(\text{_____}, \text{_____}, \text{_____})$$