

Math Q114 Exponential Function Review Problems

<p><b>Exponential Growth</b>  <math>y=Ca^x</math>                      C = initial value (vertical intercept)  <math>a &gt; 1</math> exponential growth                      a = growth factor:</p>	<p><b>Exponential Growth</b>  <math>y =C(1+r)^x</math>                      C = initial value (vertical intercept)  <math>1+ r</math> is the growth factor and r (in decimal form) is growth rate</p>
<p><b>Exponential Decay</b>  <math>y=Ca^x</math>                      C = initial value (vertical intercept)  <math>0&lt;a&lt;1</math> exponential decay                      a = decay factor:</p>	<p><b>Exponential Decay</b>  <math>y =C(1- r)^x</math>                      C = initial value (vertical intercept)  <math>1 - r</math> is the decay factor and r (in decimal form) is decay rate</p>

1. A extremely toxic pollutant was just dumped into a nearby lake. The polluters were fined and a clean-up process has been started. It is estimated that each year in the future the amount of pollutant in the lake will be reduced by 20%. The original amount that was dumped into the lake was 1200 grams. Use this information to answer the following three questions.

- Find the amount of pollutant that will be in the lake 4 years after the original dumping. Be sure to show your work. Be sure to give the units of your answer.
- Construct an equation to describe the amount of pollutant still in the lake after t years. Clearly indicate what the dependent and independent variables are and what they represent and remember to indicate their units.
- The EPA says that the lake will be safe when the toxic pollutant is below 10 grams. How long will it take for that to happen? Explain clearly how you obtained your answer. Be sure to give units for each of your answer.

2. In 1999, a worker's retirement account was valued at \$22,000. This money was invested in a Roth IRA that has grown exponentially at the rate of 14% annually.

- If this type of growth continues, find the estimated value of the retirement account in 2009.
- If this type of growth continues, estimate when the value of the retirement account will reach \$100,000 (you should use Excel or trial and error).
- Find an equation giving the value of the retirement account as a function of years. Define your variables, including units:
  - Independent variable
  - Dependent variable
  - Equation



## Linear vs Exponential Functions

<p><b>Linear Growth</b>  <math>y = mx + b</math>  <math>b</math> = initial value (vertical intercept)  <math>m</math> = average rate of change or slope  <i>If growth given as rate of change use linear model</i></p>	<p><b>Exponential Growth</b>  <math>y = C(1+r)^x</math>  <math>C</math> = initial value (vertical intercept)  <math>1+r</math> is the growth factor and <math>r</math> (in decimal form) is growth rate  <i>If growth given as % change use exponential model</i></p>
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3. Fraternal twins Jocasta and Wanda were each given a sum of \$500 when they graduated from high school. Jocasta invested her entire gift in a money market account that earned 5%, compounded annually. Wanda placed her gift in a secure account that earned \$30 each year.
- a. Write an equation for each investment. Let  $x$  be years, and  $y$  be the value in dollars.
  - b. Make an Excel spreadsheet to and graph to show the value of both Jocasta's and Wanda's investments over a 20 year period.
  - c. Write a 60 second summary comparing the sisters' earnings from their investments. (Who made the better choice? Does the period of time influence your answer to this question?)
4. A wealthy but frugal friend wants to purchase a new car that will retain its value over time. She can purchase a new Lexus for \$42,000 that declines in value 11% annually. Alternately, she can purchase a Volvo for \$30,000 that decreases in value \$1250 each year. She has come to you for advice.
- a. Construct an Excel table of the comparative values of the two cars over a time period of 15 years. Then construct a scatterplot of the values.
  - b. Write an equation for the value of each car over time.
  - c. What is your advice to your friend about which car to buy?



## Comparing Exponential Rates of Growth/Decay

5. Carlos, who is 25 years old, has rolled over \$10,000 from the retirement account at his previous job, which he plans to invest in mutual funds that earn 4% annually. Angelina, also 25, has \$5,000 to invest toward retirement. She chooses a high-yield bond that earns 8% interest annually.

- a. What is the projected the values of their investments when they come to retire at age 65?
- b. Write an exponential equation for the Value  $V$  of each of their investments over  $t$  years.
- c. Using Excel, create a spreadsheet showing the value for both Carlos and Angelina's investment for 40 years, then create a scatterplot showing both on the same graph.
- d. Carlos believes that he will be in much better shape financially than Angelina when it comes time to retire because he was able to invest twice as much money at the start. Angelina disagreed. "You may be better off in the short term," she told him, "but in the long run I'll be better off." Do you agree with her? Explain why or why not and back up your answer with reference to the graphs and tables you have created in Excel.



## Comparing Linear Rates of Change

6. The top speed a standard snowplow can travel on dry pavement is 30 miles per hour. Its top speed decreases by 0.85 miles per hour with each inch of snow on the highway. A new company named CleanSweep™ is trying to sell the city of Buffalo, New York, a modified snowplow with tractor-tread wheels that it claims is more efficient. The CleanSweep plow has a top speed of only 25 mph, but it's speed decreases by only 0.5 mph for each inch of snow on the highway. Buffalo is known for its heavy snowfall, sometimes accumulating 4-5 feet overnight. Which plow would you recommend to the city's Department of Public Works?

- Write an equation to describe the relation between plow speed and snow depth for each plow type. Let  $S$  be the speed and  $d$  be the snow depth.
- What is the independent variable (input)? \_\_\_\_\_
- What is the dependent variable(output)? \_\_\_\_\_
- Use Excel to make a graph of plow speed versus snow depth for each plow type.

7. Computer company Alpha is offering the following deal: For only \$399, you can buy a brand-new computer. However, you must also buy their internet service, which costs \$29.99 per month. Computer company Beta offers the same computer for lease for \$49.99 per month.

a) Write an equation that describes the amount of money you pay based on the number of months of internet service that you buy. Also write an equation that describes the amount of money you pay based on the number of months you lease the computer from Beta.

b) Identify each of the following in your equation, including units.

Independent variable: \_\_\_\_\_

Dependent variable: \_\_\_\_\_

Slope: \_\_\_\_\_

Vertical intercept: \_\_\_\_\_

c) Graph the Alpha and the Beta plans .

d) The fine print says that you must buy at least 24 months of internet service. With this in mind, which computer offer is the better deal?



