Math Q114 Supplementary Exercises **Name:**

Introduction to Scientific Notation

Formally, the definition of scientific notation is that numbers are written $N \times 10^n$ where $1 \le |N| < 10$. We will call the number N the "seed" of the number.

Exercise

1. Express each of the following numbers in scientific notation.

   a) 3,000 = _____

   b) 230,000 = _____

   c) 531.6 = _____

   d) –45,100 = _____

   e) 0.007 = _____

   f) –0.000045 = _____

   g) 2.3 billion = _____

Scientific Notation to Standard Notation

We can also write numbers that are in scientific notation as standard numbers. Write each of the following in standard notation:

Exercise

2. Express each of the following numbers in standard notation.

   a) 1.4×10^7 = _____

   b) -2.34×10^{12} = _____

   c) 5.8×10^{-4} = _____

   d) -8.52×10^{-7} = _____

   e) 4.7231×10^2 = _____

   f) 10^{-10} = _____

"Almost" Scientific Notation

Sometimes numbers appear to be written in scientific notation when in fact they aren't because the "seed" is not between 1 and 10. In this case we have to rewrite the "seed" in scientific notation and then use properties of exponents ($10^a \times 10^b = 10^{a+b}$) to combine the powers of ten.

Exercise

3. Write each of the following numbers in scientific notation:

   a) 32.6×10^6 = _____

   b) 0.534×10^3 = _____

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c) $125.8 \times 10^{-8} =$ _____

d) $0.041 \times 10^{-3} =$ _____

Scientific Notation on a Calculator

Many calculators also use scientific notation to store and represent large and small numbers. Use the calculator on the Mac (from the apple menu) to perform the following operations. Write down the calculator display and then convert this number to scientific notation.

4. 123,456 x 87,654,321 = _____ = _____

5. 567 ÷ 123,456,789 = _____ = _____

Calculations Involving Scientific Notation

We can also use scientific notation to perform mathematical calculations with large and small numbers. Follow these steps to multiply and divide large and small numbers.

- First convert the numbers to scientific notation.
- MULTIPLYING: multiply the "seeds" together and *add* the exponents ($10^a \times 10^b = 10^{a+b}$).
- DIVIDING: divide the "seeds" and *subtract* the exponents ($10^a \div 10^b = 10^{a-b}$).
- If the answer is in proper scientific notation (i.e. the "seed" is between 1 and 10), then you are done. If not then you need to rewrite the number with a "seed" between 1 and 10.

Exercise

6. Calculate each product or quotient and express the final answer in scientific notation.
   a) $(4.72 \times 10^{19}) \times (5.9 \times 10^{3}) =$

   b) $(8.4 \times 10^{17}) \div (2.1 \times 10^{14}) =$

   c) $(2.2 \times 10^{4}) \times (3.1 \times 10^{11}) \times (5.3 \times 10^{-6}) =$

   d) $(0.000000032) \times (250{,}000) \div (0.0016) =$

   e) $(4.3 \times 10^{7})^{3} =$

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Putting Numbers in Perspective

We often read about very large and very small numbers but do we have a really good intuition about what these numbers actually represent? To understand numbers better it is often useful to put them into perspective. We will discuss 6 different techniques for "putting numbers in perspective". These are: estimation by rounding, ratios, orders of magnitude, scale models, "guess-timation", and creative conversions. There is no single method that works best for all numbers. The "best" method will often depend on the context.

Estimation By Rounding

Numbers can sometimes be reported to a greater level of precision than we need. We may only want a "ballpark" idea of the result. A reasonable estimate can often be obtained by rounding the numbers involved before performing the required calculation. It is often adequate to round to a single significant digit and to express the numbers in scientific notation before making any calculations.

Example

The U.S. gross national debt in 2003 was $6,914,000,000,000 and the U.S. population as of July 1, 2003 was 290,810,000. If we want to determine each person's share of the national debt we would need to divide the debt by the U.S. population.

$$\$6{,}914{,}000{,}000{,}000 \div 290{,}810{,}000 = \$23{,}774.97 \text{ per person.}$$

Note that this calculation is very precise however it is probably not very accurate. Why not?

In this circumstance it would be adequate to get an estimate. If we round each number to one significant digit and express them in scientific notation, we get:

$$\$7 \times 10^{12} \div 3 \times 10^{8} = \$2.3333 \times 10^{4} = \$23{,}333$$

Note that this estimate is reasonably close to the exact answer to be acceptable.

Exercise

7. Estimate each of the following values by rounding each number to one significant digit. Also determine the exact value using a calculator.
   a) (582,015) ÷ (3,481) =

   b) (32,412,000) x(183,670) =

   c) (0.000000005612) x (7,210,650) =

   d) (23,560) ÷ (0.00006241) =

8. In 2000, Americans spent $65 billion on illegal drugs. The U.S. population as of the April 1, 2000 census was 281,421,906. Estimate the amount spent on illegal drugs in 2000 by each person in the U.S. Is this number a reasonable estimate of how much each person spends on illegal drugs?

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9. The basic "yardstick" that astronomers use to measure vast distances between stars and galaxies is the Light Year, or LY. A light year is the distance light can travels in one year. One LY = 5.88×10^{12} miles. The nearest star to our Sun is Alpha Centauri. It is 4.3 light years distant. To calculate how many miles away Alpha Centauri is we would multiply using scientific notation $(4.3 \times 10^{0})(5.88 \times 10^{12}$ miles) = $(4.3 \times 5.88)(10^{12} \times 10^{0}) = 25.284 \times 10^{12} = 2.5284 \times 10^{1} \times 10^{12} = 2.5284 \times 10^{13}$ miles.

   Write each distance below in scientific notation:

   a. The Horsehead Nebula is 1,500 LY from Earth. How many miles distant is it?

   b. The Milky Way galaxy in which our Sun is one of millions of stars, is a flat disk 120,000 LY in diameter. Express the diameter of the Milky Way galaxy in miles.

   c. The Earth is situated 30,000 LY from the center of the Milky Way galaxy. How far is that expressed in miles?

   d. The bright red star Betelguese in the belt of the constellation Orion is 650 LY distant from Earth. How far is that expressed in miles?

10. Estimate how many seconds you have lived by age 20?

11. The closest distance between Earth and the Moon is 360,000,000 meters. If an average adult is 1.68 meters tall, how estimate how many adults standing on one another's shoulders it would take to reach the Moon

12. A dollar bill is 0.0043 inches thick. Estimate how high (in feet) a stack of dollar bills equal to the federal deficit would be. (The October 2004 federal deficit is $7.4 trillion.

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