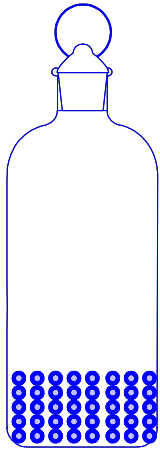
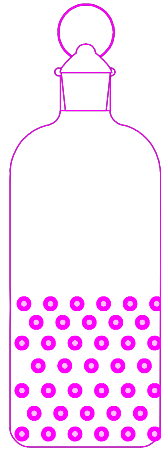


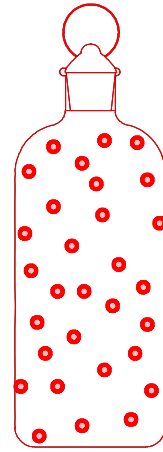
States of Matter



Solid



Liquid



Gas

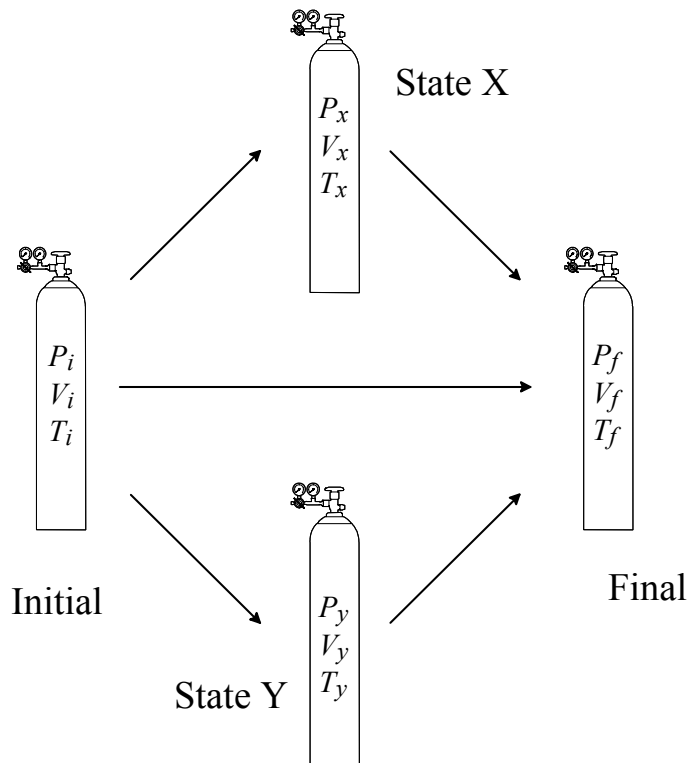
Condensed States

State Functions

The state of a certain amount of gas is specified by three inter-related variables, called **state functions**:

P - Pressure V - Volume T - Temperature

- Values of P , V , and T depend on the state of the gas, regardless of how the state was achieved.



- For a given sample, specifying two variables fixes the third.
- Relationships between P , V , and T are **equations of state**, commonly called the **gas laws**.

Pressure

- L Pressure is defined as a force per unit area:

$$P = F/A = ma/A$$

where F is force, A is area, m is mass, and a is acceleration.

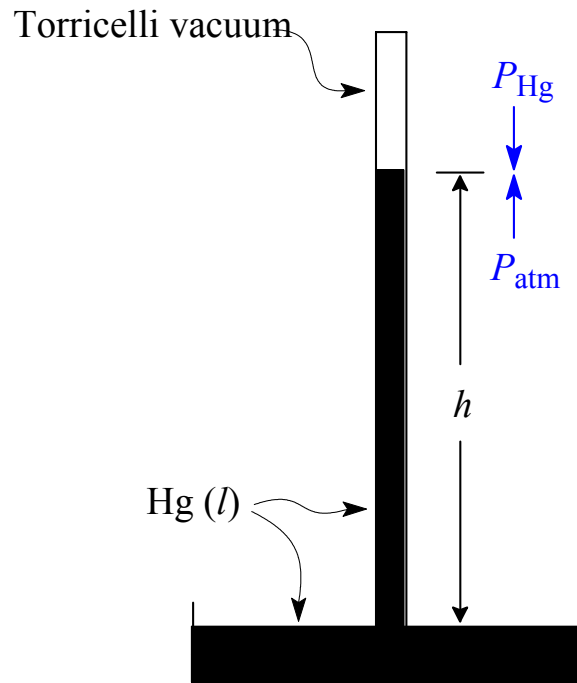
- L The units of pressure are a result of this defining equation:

$$P_{\text{cgs}} = (\text{g})(\text{cm}\cdot\text{s}^{-2})/\text{cm}^2 = \text{dyne}/\text{cm}^2$$

$$P_{\text{SI}} = (\text{kg})(\text{m}\cdot\text{s}^{-2})/\text{m}^2 = \text{newton}/\text{m}^2 = \text{pascal (Pa)}$$

Barometer

Evangelista Torricelli - 1643



- L Liquid mercury in the tube falls to a level at which its downward pressure equals the counter-balancing pressure exerted by the air on the surface of the mercury pool.

$$P_{\text{atm}} = P_{\text{Hg}}$$

- L The pressure of the mercury column is the product of the acceleration of gravity (g), the density of the mercury (d), and the height of the column (h):

$$P_{\text{Hg}} = gdh = P_{\text{atm}}$$

- L Both g and d are constants, so

$$P_{\text{atm}} \propto h$$

Pressure Units Related to the Height of a Mercury Column

- The height of the mercury column is usually measured in millimeters, abbreviated

mm Hg

- A pressure equivalent to a millimeter of mercury is called a **torr** (in honor of Torricelli):

1 mm Hg / 1 torr

- Standard atmospheric pressure (abbreviated atm) is the barometric pressure that sustains a mercury column of exactly 760 mm:

1 atm / 760 mm Hg / 760 torr

- Nowadays, the standard atmosphere has been redefined in terms of the pascal:

1 atm / 101,325 Pa (exactly)

- The SI unit comparable to atm is the bar = 10^5 Pa:

1 atm / 1.01325 bar (exactly)

Manometers

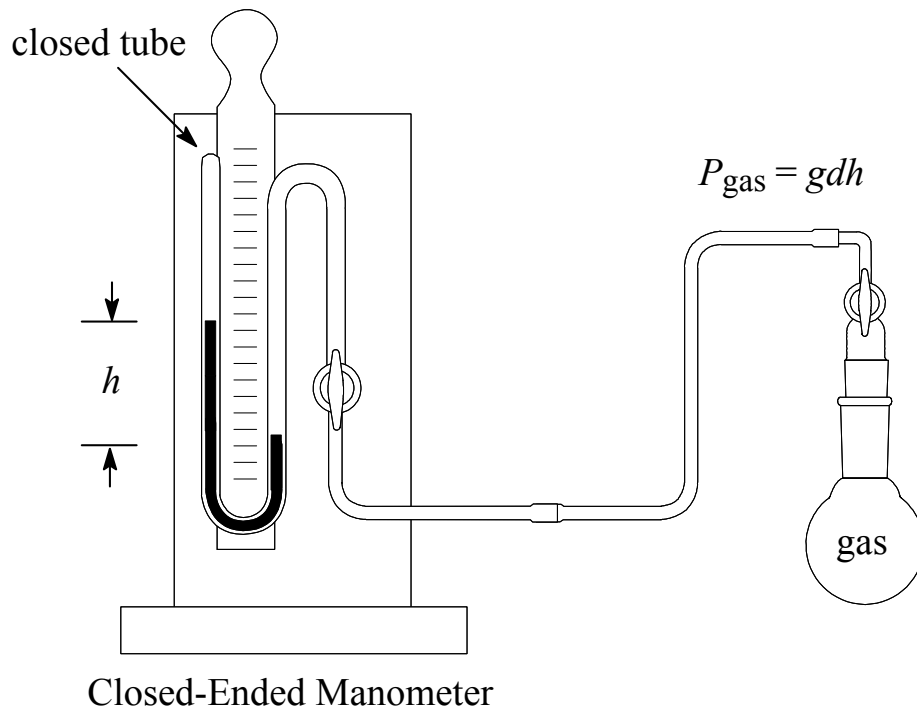
- Pressures of gas samples are routinely measured with a modification of the barometer, called a **manometer**.
 - Closed-ended manometers measure gas-sample pressure independent of atmospheric conditions.

$$P_{\text{gas}} = \rho gh$$

- Open-ended manometers measure gas-sample pressure relative to the room (ambient) pressure, which must be measured.

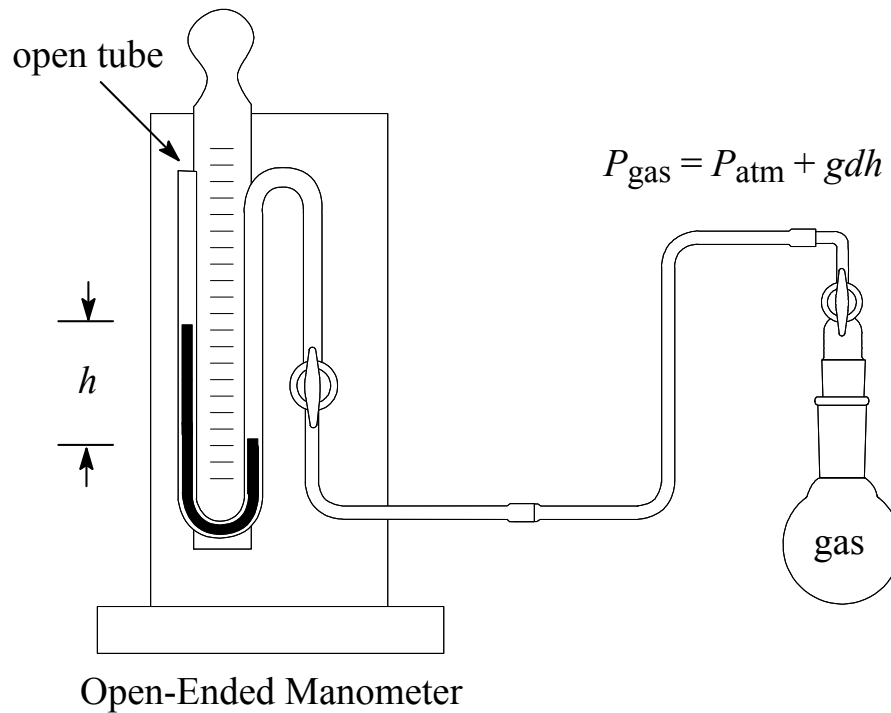
$$P_{\text{gas}} = P_{\text{atm}} \pm \rho gh$$

Closed-Ended Manometer



- L Difference in height between the two sides indicates the sample gas pressure.

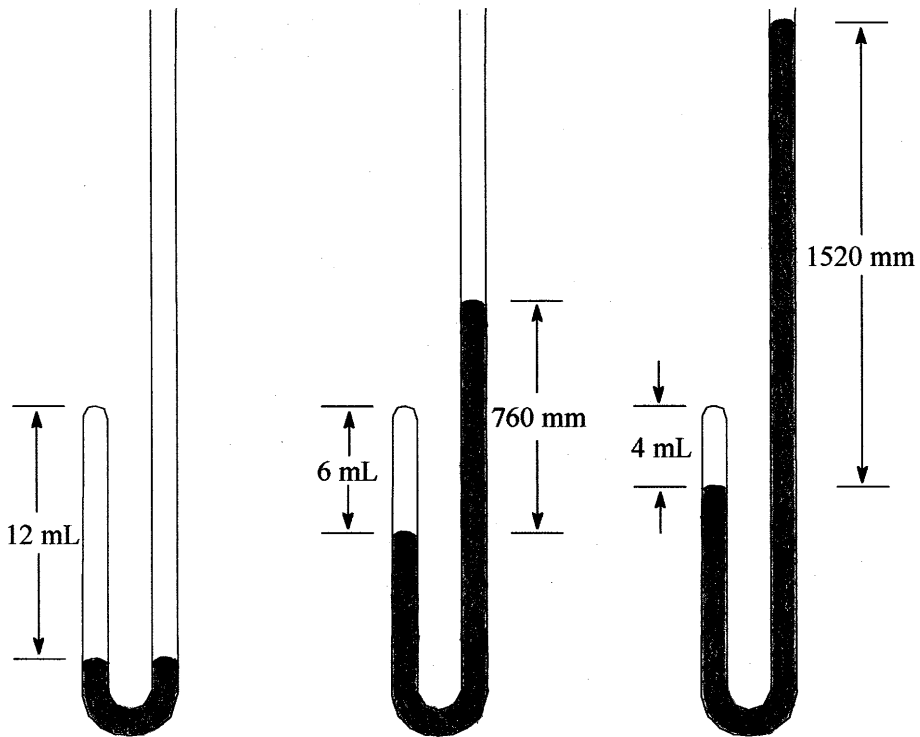
Open-Ended Manometer



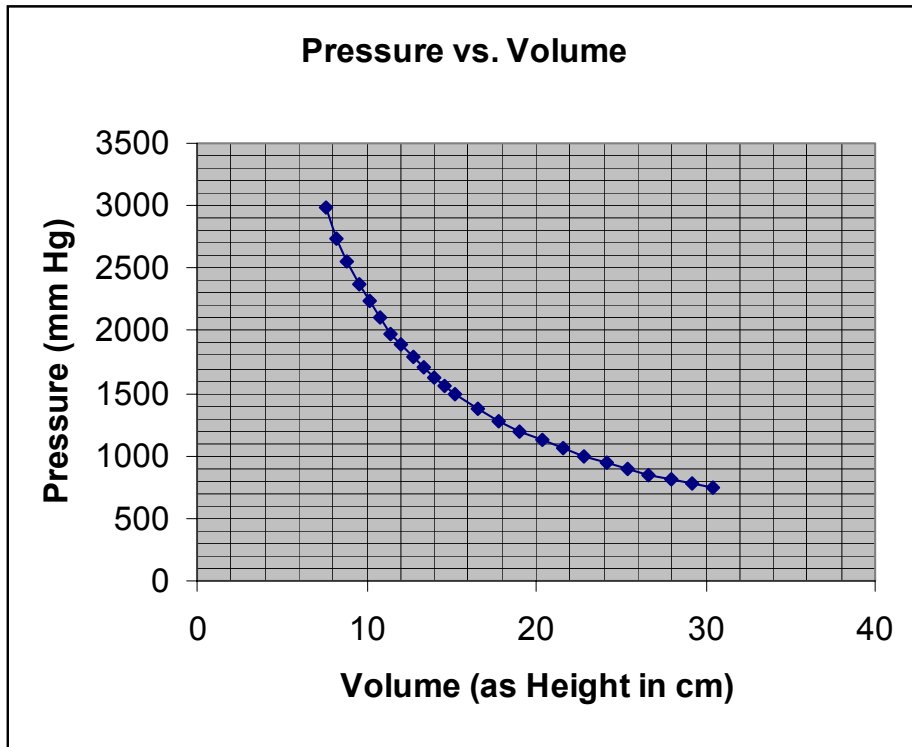
- L If the height of the outer arm is higher than the inner arm, $P_{\text{gas}} > P_{\text{atm}}$ and $P_{\text{gas}} = P_{\text{atm}} + gdh$.
- L If the height of the inner arm is higher than the outer arm, $P_{\text{gas}} < P_{\text{atm}}$ and $P_{\text{gas}} = P_{\text{atm}} - gdh$.

Robert Boyle's Pressure-Volume Experiments 1662

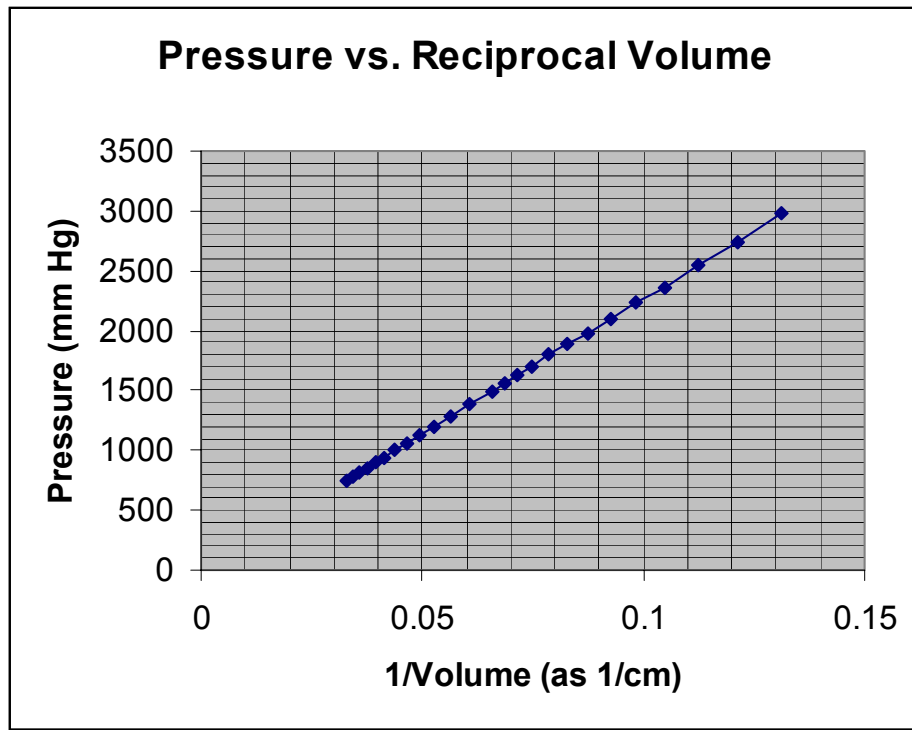
$$P_1 = P_{\text{atm}} = 760 \text{ torr} \quad P_2 = 1520 \text{ torr} \quad P_3 = 2280 \text{ torr}$$



Historical Data from Boyle's Pressure vs. Volume Experiments



Boyle's Data Replotted as Pressure vs. Reciprocal Volume



Boyle's Law

- L For a fixed amount of gas at constant temperature, volume is inversely proportional to pressure.

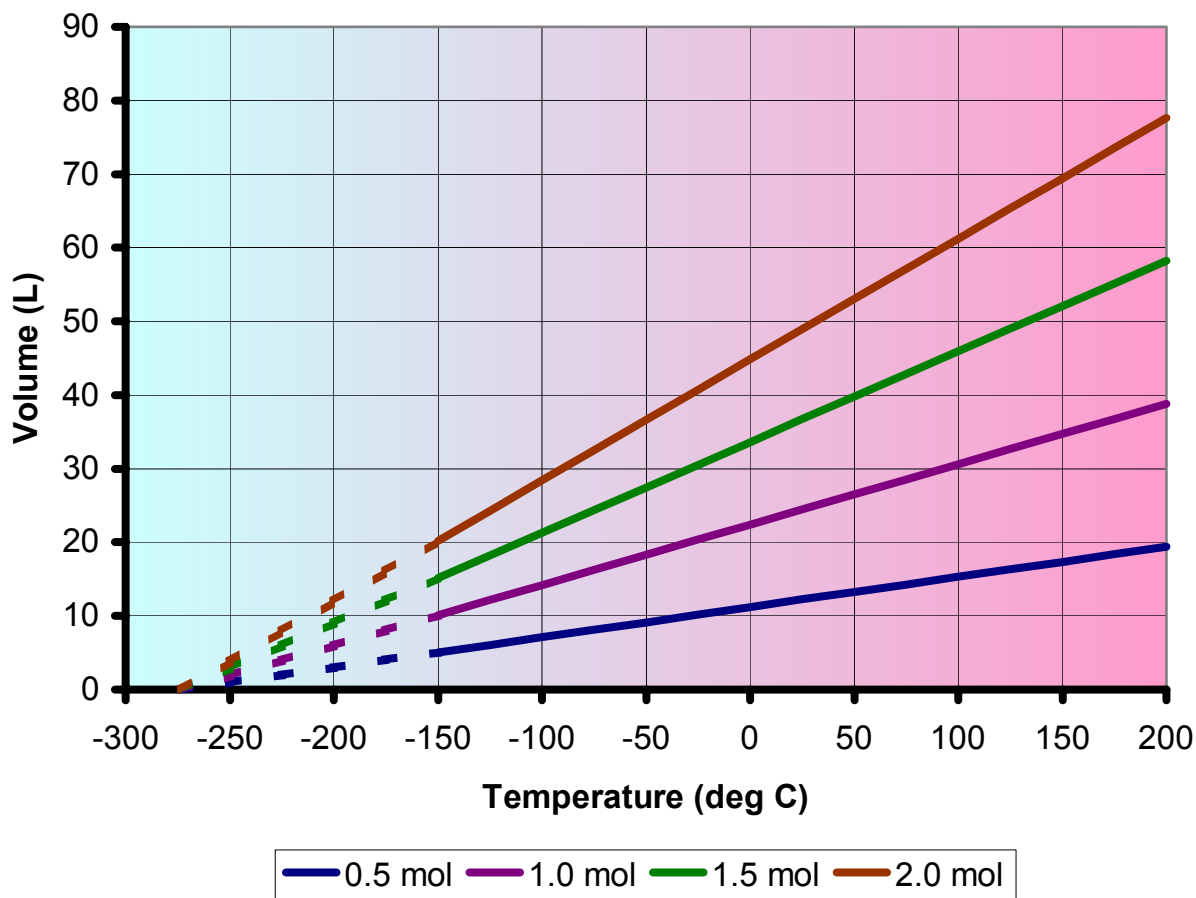
$$V \propto 1/P$$

$$V = b/P \quad VP = b \quad b = f(n, T)$$

$$P_1V_1 = P_2V_2$$

Jacques Charles' Temperature-Volume Experiments 1787

Volume vs. Temperature for Various Amounts of Gas
at 1 atm



Charles' Law

- L For a fixed amount of gas at constant pressure, volume is proportional to *absolute* temperature.

$$V = cT \quad c = f(n, P)$$

- K Note: Temperature must be in Kelvin (K)!

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Combined Gas Law

- L Combining Boyle's and Charles' Laws for a fixed amount of gas:

$$V \propto \frac{T}{P} \quad \text{or} \quad V = k \left(\frac{T}{P} \right)$$

$$\frac{PV}{T} = k \quad \text{or} \quad k = f(n)$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

**Combined Gas Law Equation
to
Ideal Gas Equation of State
(Ideal Gas Law Equation)**

From the Combined Gas Law:

$$\frac{PV}{T} \propto k \quad k \propto f(n)$$

If $k \propto n$, then we can write an equation for k by inserting a proportionality constant, which we will call R :

$$k = nR$$

Substituting into $PV/T = k$ gives $PV/T = nR$, or

$$***PV = nRT***$$

- L $PV = nRT$ is the **equation of state of an ideal gas**, also called the **ideal gas law** equation.
- L $PV = nRT$ is the most important of all the fundamental gas law equations.

Ideal Gas Law

$$PV = nRT$$

$$R = 0.08206 \text{ L}\cdot\text{atm}/\text{K}\cdot\text{mol}$$

- L R , the **gas law constant**, is a fundamental constant of the Universe and appears in many important physical equations, in addition to $PV = nRT$.
- L The value of R depends upon the units used.

Units	Numerical value
L·atm/K·mol	0.08206
J/K·mol	8.314
cal/K·mol	1.987
L·torr/K·mol	62.36

t R values we will use are in blue.

Variations on $PV = nRT$

Let m = mass of sample and M = molecular weight. Then the number of moles of gas is

$$n = m/M$$

Substituting into $PV = nRT$ gives

$$PV = \left(\frac{m}{M} \right) RT \quad \text{Y} \quad M = \frac{mRT}{PV}$$

Let d = density = m/V :

$$M = \left(\frac{m}{V} \right) \frac{RT}{P} = \frac{dRT}{P}$$

$$\text{Y} \quad d = \frac{PM}{RT}$$

Law of Amontons

Guillaume Amontons (Fr., 1663 - 1705), c. 1703

- L For a sample of gas in a fixed volume, the pressure is directly proportional to the absolute temperature.

Let the volume for a certain gas sample be fixed; i.e., V , n , and R are constant. Then from $PV = nRT$, gathering the variables on the left and the constants on the right

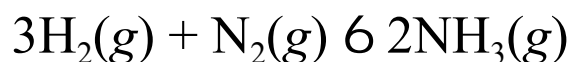
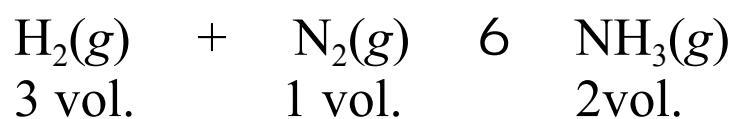
$$\frac{T}{P} = \frac{V}{nR} = a \quad a = f(n, V)$$

$$\frac{T_1}{P_1} = \frac{T_2}{P_2}$$

Gay-Lussac's Law of Combining Gas Volumes

Joseph Louis Gay-Lussac (Fr., 1778-1850), 1808

- L In reactions between gases at constant temperature and pressure, the volumes that react are in the ratios of small whole numbers.



**Gay-Lussac's
Law of Combining Gas Volumes
from $PV = nRT$**

If P and T are held constant, then from $PV = nRT$:

$$V \propto n \left(\frac{RT}{P} \right) \quad \text{or} \quad V \propto n \quad \text{or} \quad V \propto f(P, T)$$

Avogadro's Hypothesis

Amedeo Avogadro (It., 1776 - 1856), 1811

- L Equal volumes of all gases at the same temperature and pressure contain the same number of molecules.

If P and T are held constant, then from $PV = nRT$:

$$V \propto n \left(\frac{RT}{P} \right) \quad \text{or} \quad V \propto n \quad \text{or} \quad V \propto f(P, T)$$

**Standard Temperature (0 °C)
and
Pressure (1.00 atm)
(STP)**

At STP one mole of ideal gas occupies
22.4 L
called the *molar volume* of an ideal gas at STP.

$$1 \text{ mol} = 22.4 \text{ L at STP}$$

Gas Law Summary

$$PV = nRT$$

Boyle: $PV = nRT$ V, P variable n, T constant
 $P_1V_1 = P_2V_2$

Charles: $PV = nRT$ V, T variable n, P constant
 $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

Amontons: $PV = nRT$ P, T variable n, V constant
 $\frac{T_1}{P_1} = \frac{T_2}{P_2}$

Gay-Lussac & Avogadro: $PV = nRT$ V, n variable P, T constant
 $\frac{n_1}{V_1} = \frac{n_2}{V_2}$

Other: $PV = nRT$ P, n variable T, V constant
 $\frac{n_1}{P_1} = \frac{n_2}{P_2}$

General: $PV = nRT$ P, V, T variable n constant
 $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$