## Weak Base in Water

For a solution of a weak base in water, the determination of all species concentrations follows analogously with the acid case; i.e., four equations in four unknowns to which simplifications can often be made.
(1)

$$
\begin{gathered}
\mathrm{B}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{BH}^{+}+\mathrm{OH}^{-} \\
2 \mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{OH}^{-}
\end{gathered}
$$

$$
K_{b}=\frac{\left[\mathrm{BH}^{+}\right]\left[\mathrm{OH}^{-}\right]}{[\mathrm{B}]}
$$

$$
\begin{equation*}
K_{w}=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{OH}^{-}\right]=1.00 \times 10^{-14} \tag{2}
\end{equation*}
$$

The charge balance expression is
(3) $\left[\mathrm{OH}^{-}\right]=\left[\mathrm{OH}^{-}\right]_{\text {base }}+\left[\mathrm{OH}^{-}\right]_{\text {water }}=\left[\mathrm{BH}^{+}\right]+\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$

The mass balance expression is

$$
\begin{equation*}
[\mathrm{B}]=C_{\mathrm{B}}-\left[\mathrm{BH}^{+}\right] \tag{4}
\end{equation*}
$$

# Weak Base In Water <br> Simplifying Assumptions 

$$
\begin{gathered}
\mathrm{B}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{BH}^{+}+\mathrm{OH}^{-} \\
2 \mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{OH}^{-}
\end{gathered}
$$

Assumption I: If $C_{\mathrm{B}} \gg 10^{-7} \mathrm{M}$, assume virtually all $\mathrm{OH}^{-}$comes from B. Thus,

$$
\left[\mathrm{OH}^{-}\right] \approx\left[\mathrm{HB}^{+}\right]
$$

Assumption II: If $C_{\mathrm{B}} \gg K_{b}$ (usually 2 powers of 10 or more), assume B is negligibly hydrolyzed. Thus,

$$
[\mathrm{B}] \approx C_{\mathrm{B}}
$$

If both assumptions can be made:

$$
K_{b}=\frac{\left[\mathrm{OH}^{-}\right]\left[\mathrm{BH}^{+}\right]}{[\mathrm{B}]}=\frac{\left[\mathrm{OH}^{-}\right]^{2}}{C_{\mathrm{B}}}
$$

and

$$
\left[\mathrm{OH}^{-}\right]=\sqrt{C_{\mathrm{B}} K_{b}}
$$

## Breakdown of Assumption II

If $C_{\mathrm{B}} \approx K_{b}$ (usually within 2 powers of 10 ), assume B is not negligibly hydrolyzed. Thus,

$$
\begin{gathered}
{[\mathrm{B}]=C_{\mathrm{B}}-\left[\mathrm{OH}^{-}\right]} \\
K_{b}=\frac{\left[\mathrm{OH}^{-}\right]\left[\mathrm{BH}^{+}\right]}{[\mathrm{B}]}=\frac{\left[\mathrm{OH}^{-}\right]^{2}}{C_{\mathrm{B}}-\left[\mathrm{OH}^{-}\right]}
\end{gathered}
$$

$$
\left[\mathrm{OH}^{-}\right]^{2}+K_{b}\left[\mathrm{OH}^{-}\right]-K_{b} C_{\mathrm{B}}=0
$$

Solve the quadratic equation for $\left[\mathrm{OH}^{-}\right]=\left[\mathrm{BH}^{+}\right]$

## $K_{a}$ 's of Polyprotic Acids

- We can define a $K_{a}$ for each dissociation of a polyprotic acid.

$$
\begin{array}{ll}
\mathrm{H}_{3} \mathrm{PO}_{4}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{H}_{2} \mathrm{PO}_{4}^{-} & K_{1}=7.52 \times 10^{-3} \\
\mathrm{H}_{2} \mathrm{PO}_{4}^{-}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{HPO}_{4}^{2-} & K_{2}=6.23 \times 10^{-8} \\
\mathrm{HPO}_{4}^{2-}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{PO}_{4}^{3-} & K_{3}=4.5 \times 10^{-13}
\end{array}
$$

- As is typical of polyprotic acids, $K_{1}>K_{2}>K_{3}$, etc.


## Determining $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right.$] for a Polyprotic Acid

- All equilibria occur simultaneously, but $K_{1}$ is usually so much greater than $K_{2}$ and $K_{3}$ that in a moderately concentrated solution we can assume that virtually all of the hydronium ion comes from the first step.
$\checkmark$ Charge balance expression:

$$
\begin{aligned}
{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] } & =\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]_{1}+\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]_{2}+\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]_{3}+\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]_{\text {water }} \\
& =\left[\mathrm{H}_{2} \mathrm{PO}_{4}^{-}\right]+\left[\mathrm{HPO}_{4}^{2-}\right]+\left[\mathrm{PO}_{4}^{3-}\right]+\left[\mathrm{OH}^{-}\right] \\
& \approx\left[\mathrm{H}_{2} \mathrm{PO}_{4}^{-}\right]
\end{aligned}
$$

$\checkmark$ For calculations of a pure polyprotic acid in water (with no additional conjugate base) we only need to consider $K_{1}$ to find $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right.$.
$\checkmark$ For polyprotic acids in general, this assumption is valid only if the successive $K$ 's are widely separated; i.e., $K_{1} \gg K_{2} \gg K_{3}$, etc.

## Simplifying Assumptions for a Polyprotic Acid, $\mathbf{H}_{n} \mathbf{A}$

Assumption I: If $C \gg 10^{-7} \mathrm{M}$, assume virtually all $\mathrm{H}_{3} \mathrm{O}^{+}$comes from the first dissociation. Thus, for $\mathrm{H}_{n} \mathrm{~A}$,

$$
\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \approx\left[\mathrm{H}_{(n-1)} \mathrm{A}^{-}\right]
$$

Assumption II: If $C \gg K_{1}$ (usually 2 powers of 10 or more), assume the acid is negligibly dissociated.
Then, for $\mathrm{H}_{n} \mathrm{~A}$,

$$
\left[\mathrm{H}_{n} \mathrm{~A}\right] \approx C
$$

Breakdown of Assumption II: If $C \approx K_{1}$ (usually within 2 powers of 10 ), assume the acid is not negligibly dissociated. Then, for $\mathrm{H}_{n} \mathrm{~A}$,

$$
\left[\mathrm{H}_{n} \mathrm{~A}\right]=\mathrm{C}-\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]
$$

## Calculation Procedure for a Polyprotic Acid, $\mathbf{H}_{3} \mathbf{A}$ $K_{1} \gg K_{2} \gg K_{3}$

(1) Use $K_{1}$ and Assumption I with or without Assumption II to calculate $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\left[\mathrm{H}_{2} \mathrm{~A}^{-}\right]$.
(2) $\quad\left[\mathrm{HA}^{2-}\right] \approx K_{2}$.
(3) Use $K_{3}$ and the previously calculated values of $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$and $\left[\mathrm{HA}^{2-}\right]$ to calculate $\left[\mathrm{A}^{3-}\right]$.
(4) Use $K_{w}$ and the previously calculated value of $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$ to calculate $\left[\mathrm{OH}^{-}\right]$.
(5) Check Assumption I by substituting calculated values into the exact charge balance expression to verify

$$
\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\left[\mathrm{H}_{2} \mathrm{~A}^{-}\right]+\left[\mathrm{HA}^{2-}\right]+\left[\mathrm{A}^{3-}\right]+\left[\mathrm{OH}^{-}\right] \approx\left[\mathrm{H}_{2} \mathrm{~A}^{-}\right]
$$

(6) If using Assumption II, check to make sure the error in the mass balance expression is acceptably small; i.e.,

$$
\left[\mathrm{H}_{3} \mathrm{~A}\right]=\mathrm{C}-\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \approx C
$$

$$
\text { and } \quad\left(\left[\mathrm{H}_{2} \mathrm{~A}^{-}\right] / C\right) \times 100 \%<10 \%
$$

