Chapter 8: A closer look at assumptions
Chapter 9: Multiple regression

Class 15: 4/6/09 M

Slide 1 Chapter 8: A closer look at assumptions

Chapter 9: Multiple regression

NOTES:

HW 10 due Weds 4/8/09

Submit as Myname-HW9.doc (or *.rtf)

- Computation Problem 10, chapter 8
  - 8.16 Meat processing, Must assess lack of fit!
  - Due Weds 4/8 10 am
- Read Chapter 3 from Draper & Smith on regression, especially designing a standard curve (includes lack of fit from Chapter 8)
- Read Chapter 9 on multiple regression
  - Read chapter 9 conceptual problems & solutions
  - Post a question or response about Chapter 9 conceptual problems
- HW 11 9.19: Speed of evolution,
  - Due Monday 4/13/09 10 am
  - This is a TOUGH problem! Weds: ask for help/hints!

Slide 2 HW 10 due Weds 4/8/09

NOTES:

Homework Solution Presentation

Lisa Greber: Homework 7

- Computation Problem 7
  - Problem 5.25 Duodenal ulcers

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Slide 3 Homework Solution Presentation

NOTES:

Chapter 7 (Continued) Simple Linear Regression: a model for the mean

Simple regression = ordinary least squares (OLS) regression, Model I regression

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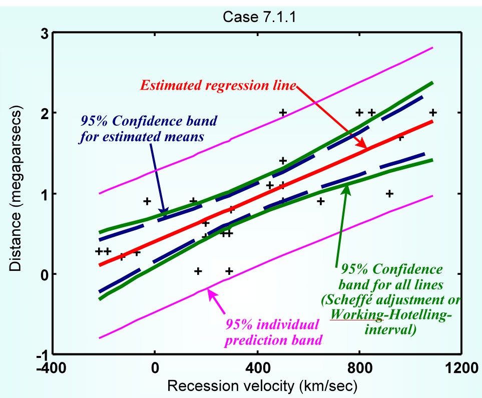
Slide 4 Chapter 7 (Continued) Simple Linear Regression: a model for the mean

NOTES:



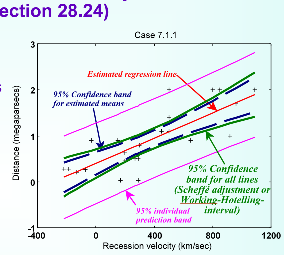
Slide 5

NOTES:

Working-Hotelling CI for a regression line

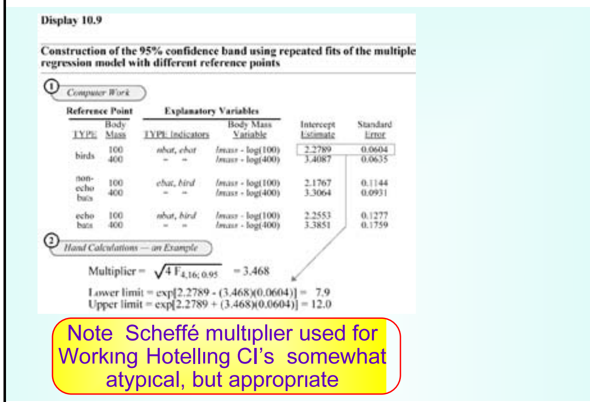
Kendall & Stuart (1979, Advanced theory of statistics, 4th edition, Section 28.24)

"Suppose now that we require a confidence region for an entire regression line, i.e. a region R in the (x,y) plane ...such that there is probability 1-α that the true regression line y=xβ is contained in R. ... we are now seeking a confidenre region, not an interval, and it covers the whole line, not one point on the line... this problem first solved in the simplest case by Working and Hotelling (1929) in a remarkable paper..." p 388



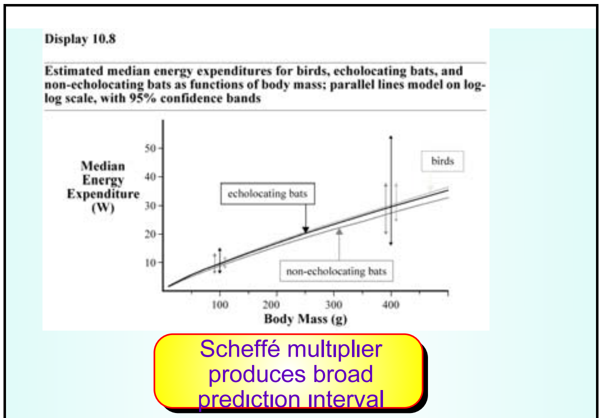
Slide 6 Working-Hotelling CI for a regression line

NOTES:



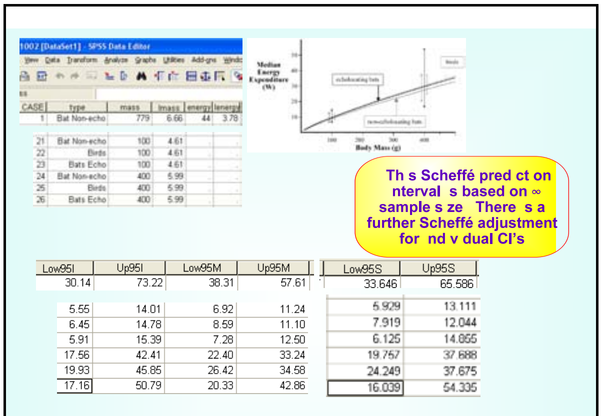
Slide 7 Display 10.9

NOTES:



Slide 8 Display 10.8

NOTES:



Slide 9

NOTES:

Syntax: Scheffé multiplier

```
* regpars is the number of parameters in the final model, with 16 df in the residual.
Compute regpars=4.
Compute residdf=16.
exe.
COMPUTE FScheffe = IDF.F(0.95,regpars,residdf) .
EXECUTE .
COMPUTE Scheffemultiplier = sqrt(regpars*FScheffe) .
EXECUTE .
* Scheffe interval is Scheffe multiplier times the standard error for each predicted value, SEP_1 was producded by regression.
COMPUTE SchInt = SEP_1 * Scheffemultiplier .
EXECUTE .
COMPUTE L95S = PRE_1  - SchInt.
COMPUTE U95S = PRE_1 + SchInt.
EXE.
COMPUTE PredE = Exp(PRE_1).
COMPUTE Low95S  = Exp(L95S) .
COMPUTE Up95S    = Exp(U95S) .
EXECUTE .
```

Slide 10 Syntax: Scheffé multiplier

NOTES:

Lack of fit tests (8.5.3) using the Regression ANOVA model

(using Case 8.2 as an example)

Chem sts & phys cal oceanographers don't use ANOVA, t s sa d They use regress on. But, they should analyze the regress on ANOVA table and test for lack of f t

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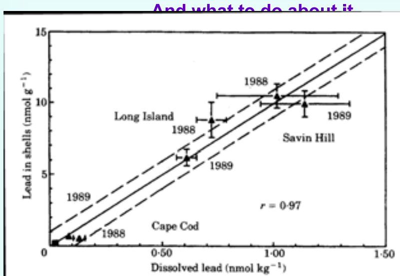
Slide 11 Lack of fit tests (8.5.3) using the Regression ANOVA model

NOTES:

How to recognize lack of fit

And what to do about it



No obvious transformation

Pitts, EEOS Ph.D.

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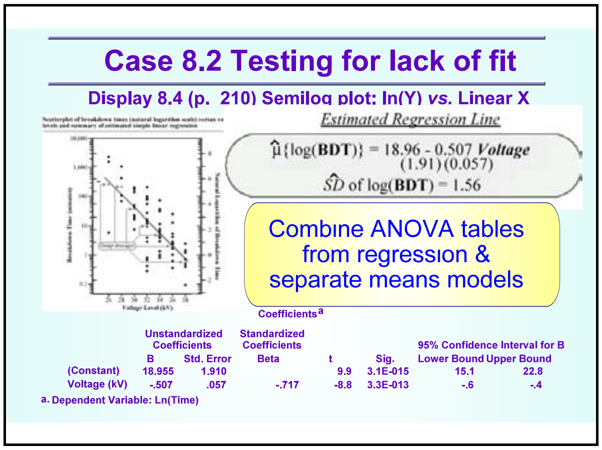
Slide 12 How to recognize lack of fit

NOTES:

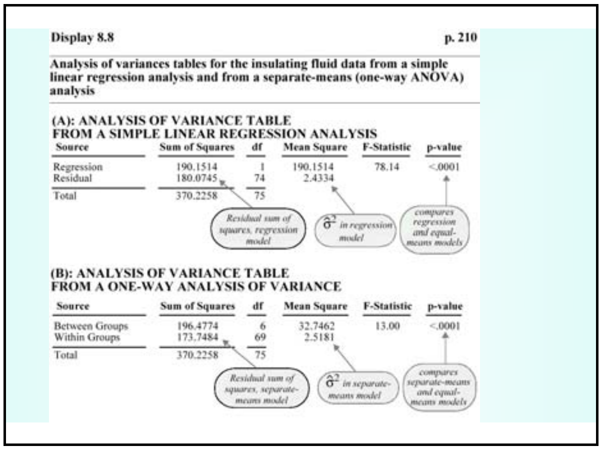


	Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
(Constant)	18.955	1.910		9.9	3.1E-015	15.1	22.8
Voltage (kV)	-.507	.057	-.717	-8.8	3.3E-013	-.6	-.4

a. Dependent Variable: Ln(Time)

Slide 13 Case 8.2 Testing for lack of fit

NOTES:


Display 8.8 p. 210

Analysis of variances tables for the insulating fluid data from a simple linear regression analysis and from a separate-means (one-way ANOVA) analysis

(A): ANALYSIS OF VARIANCE TABLE FROM A SIMPLE LINEAR REGRESSION ANALYSIS

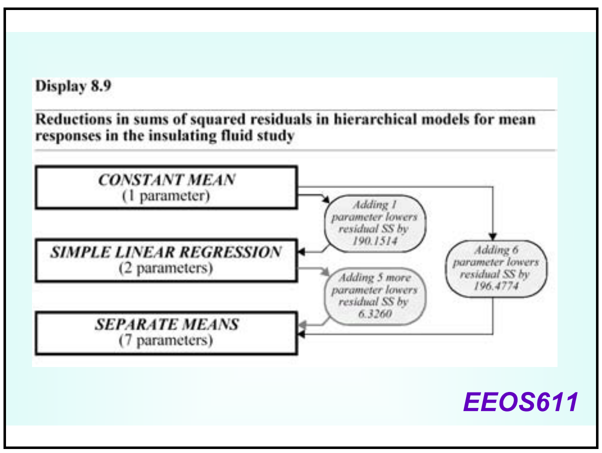
Source	Sum of Squares	df	Mean Square	F-Statistic	p-value
Regression	190.1514	1	190.1514	78.14	<.0001
Residual	180.0745	74	2.4334		
Total	370.2258	75			

(B): ANALYSIS OF VARIANCE TABLE FROM A ONE-WAY ANALYSIS OF VARIANCE

Source	Sum of Squares	df	Mean Square	F-Statistic	p-value
Between Groups	196.4774	6	32.7462	13.00	<.0001
Within Groups	173.7484	69	2.5181		
Total	370.2258	75			

Slide 14

NOTES:



Slide 15 Display 8.9

NOTES:

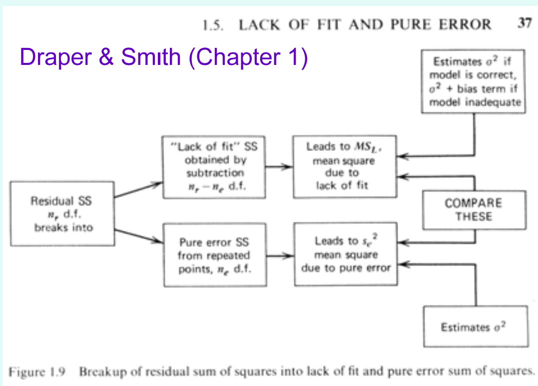


Figure 1.9 Breakup of residual sum of squares into lack of fit and pure error sum of squares.

Slide 16

NOTES:

Display 8.10

Composite analysis of variance table with F-test for lack-of-fit

Source of Variation	Sum of Squares	df	Mean Square	F-Statistic	p-value
Between Groups	*196.4774*	*6*	*32.7462*	*13.00*	*<.0001*
Regression	190.1514	1	190.1514	75.51	<.0001
Lack of Fit	**6.3260**	**5**	**1.2652**	**0.50**	**.78**
Within Groups	*173.7484*	*69*	*2.5181*		
Total	370.2258	75			

by subtraction

LEGEND
Normal type items come from Regression Analysis (A)
Italicized items come from separate-means Analysis (B)
Bold face items are new and calculated here

Residual = Pure error + Lack of fit

$$F\,stat = \frac{[SSRes_{LR} - SSRes_{SM}]\,/\,[d.f._{LR} - d.f._{SM}]}{\hat{\sigma}^2_{SM}}$$

Slide 17

NOTES:

How to test lack of fit using SPSS

1 of 2

- You must have true replicates at 1 or more values of the explanatory variables
  - [With no replicates, you can still evaluate lack of fit visually with residual plots. Curvature in residuals is a common cause of lack of fit.]
  - 1) Do ANOVAs to test for differences among means of replicated groups (Use SPSS's oneway or general linear model (UNIANOVA))
  - 2) Do a regression analysis
  - 3) Combine Regression and 'separate means' ANOVA tables
    - A) Subtract regression sum of squares (1df) from the ANOVA among group sum of squares (#replicated groups -1) to obtain 'lack of fit' sum of squares & df (# replicated groups -2)
    - B) Divide sums of squares by appropriate df and evaluate 'lack of fit' F statistic

Slide 18 How to test lack of fit using SPSS

NOTES:

How to test lack of fit using SPSS

2 of 2

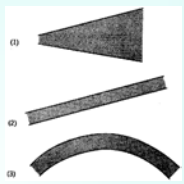
- OR, use SPSS General linear model/univariate/ options/lack of fit, enter explanatory variables as covariate
- OR, for equally spaced explanatory variables, use linear contrasts in ONEWAY or enter contrasts in oneway or general linear model
  - You can find the linear contrast values in Winer et al. (1991) or Draper & Smith or use the Matlab's orthpoly.m (in Smyth's Statbox from Matlab users' files)
  - Quadratic, cubic and quartic polynomials also provided (each set of polynomials is uncorrelated with those of lower order)

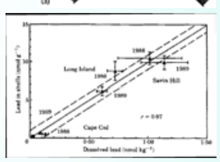
Slide 19 How to test lack of fit using SPSS

NOTES:

Strategy for dealing with lack of fit

- **You must have true replicates**
- **Examine scatterplots**
  - **Are transformations or quadratic explanatory variables needed?**
- **Fit linear regression model**
  - **Examine residuals**
  - **Transform data, add quadratic or cubic explanatory terms if needed**
  - **Add other explanatory terms (Ch 9...)**
- **Perform lack of fit test**
  - **If LOF significant with linear model, consider tests of higher order (quadratic & cubic) trends in ANOVA model**
    - Add quadratic or cubic terms to regression model if quadratic or cubic trend found
    - LOF could be due to cluster & serial effects
- **Report effect size with regression or ANOVA**
  - **Regression slope is still an unbiased estimator of true slope**
  - **Use linear contrast in ANOVA to determine effect size (GLM Unianova)**





Slide 20 Strategy for dealing with lack of fit

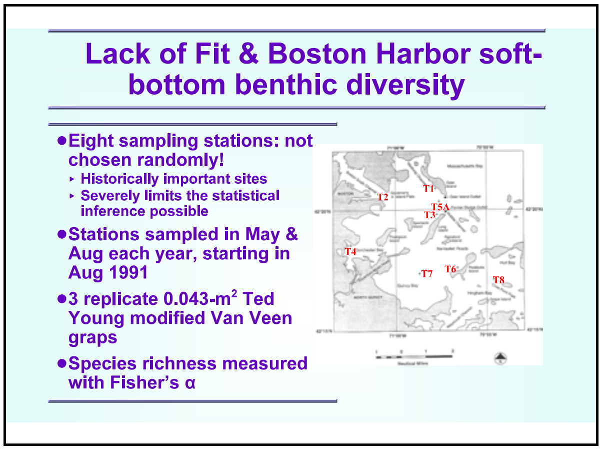
NOTES:

What to do if there is lack of fit!

- You may still estimate the slope & Y intercept using regression: OLS regression still provides unbiased estimators
  - ANOVA linear contrast will also provide estimators for linear slope & confidence intervals
- You can NOT use the variance estimates and *p* values based on the error mean square from the OLS linear regression
- Fit a richer or different model
  - Consider testing higher order interaction terms: quadratic & cubic, if warranted
  - Add other explanatory variables
- You **may** analyze the data as an ANOVA separate means model with a linear contrast (see sleuth yellow-tail fish weights as an example)

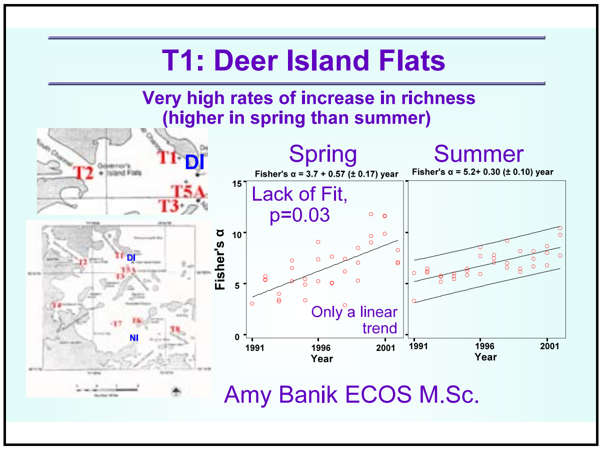
Slide 21 What to do if there is lack of fit!

NOTES:



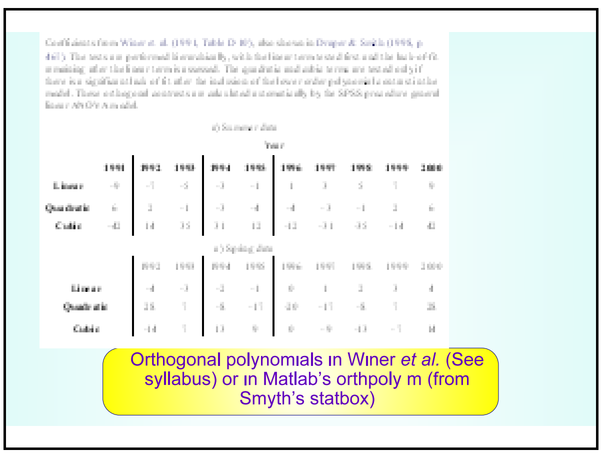
Slide 22 Lack of Fit & Boston Harbor soft-bottom benthic diversity

NOTES:



Slide 23 T1: Deer Island Flats

NOTES:



	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
Linear	-9	-7	-5	-3	-1	1	3	5	7	9
Quadratic	6	2	-1	-3	-4	-4	-3	-1	2	6
Cubic	-42	14	35	31	12	-12	-31	-35	-14	42

	1992	1993	1994	1995	1996	1997	1998	1999	2000
Linear	-4	-3	-2	-1	0	1	2	3	4
Quadratic	28	7	-8	-17	-20	-17	-8	7	28
Cubic	-14	7	13	9	0	-9	-13	-7	14

Slide 24

NOTES:

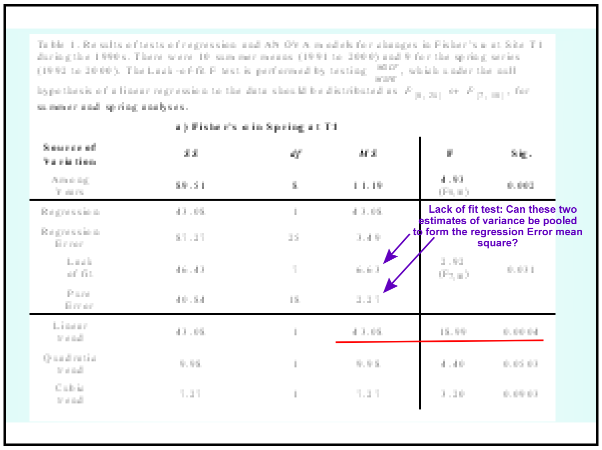


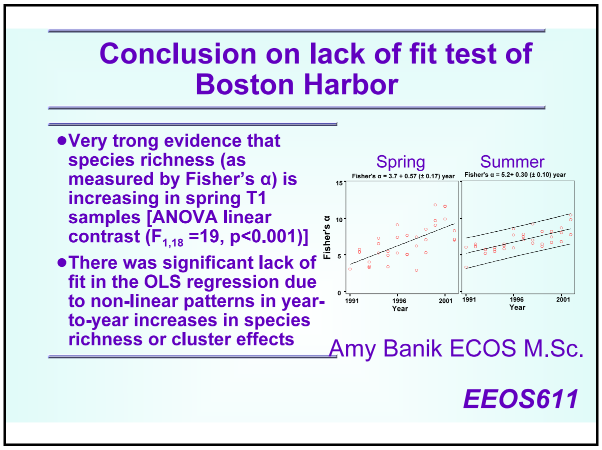
Table 1. Results of tests of regression and ANOVA models for changes in Fisher's α at Site T1 during the 1990s. There were 10 summer means (1991 to 2000) and 9 for the spring series (1992 to 2000). The Lack-of-fit F test is performed by testing [illegible], which under the null hypothesis of a linear regression to the data should be distributed as [illegible] or [illegible], for summer and spring analyses.

a) Fisher's α in Spring at T1

Source of Variation	SS	df	MS	F	Sig.
Among Years	59.51	5	11.19	4.93 ($F_{8,8}$)	0.002
Regression	43.05	1	43.05		
Regression Error	87.21	25	3.49		
Lack of fit	46.43	7	6.63	2.92 ($F_{7,18}$)	0.031
Pure Error	40.54	18	2.25		
Linear trend	43.05	1	43.05	15.99	0.0004
Quadratic trend	9.95	1	9.95	4.40	0.0503
Cubic trend	1.21	1	1.21	[illegible]	0.0903

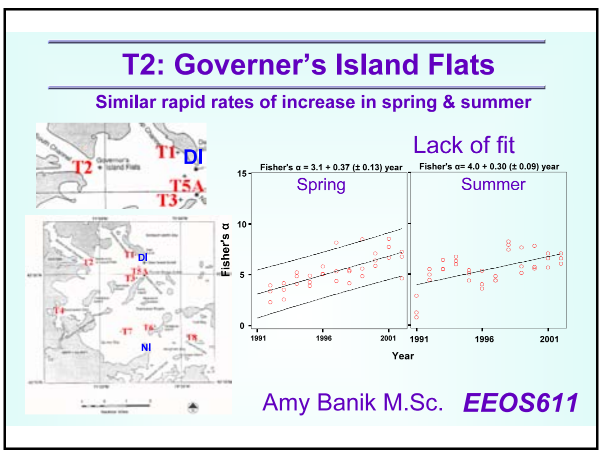
Slide 25

NOTES:



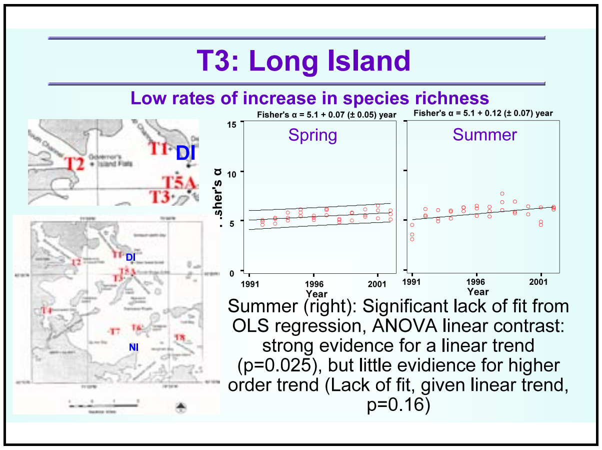
Slide 26 Conclusion on lack of fit test of Boston Harbor

NOTES:



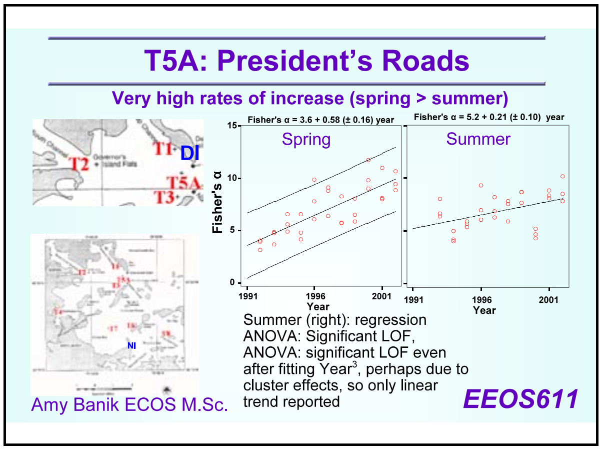
Slide 27 T2: Governer's Island Flats

NOTES:



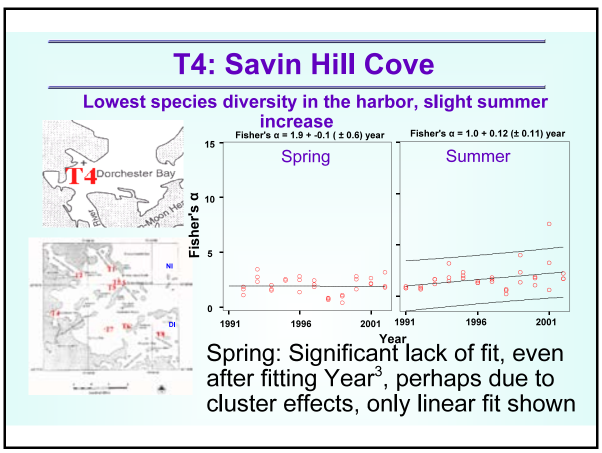
Slide 28 T3: Long Island

NOTES:



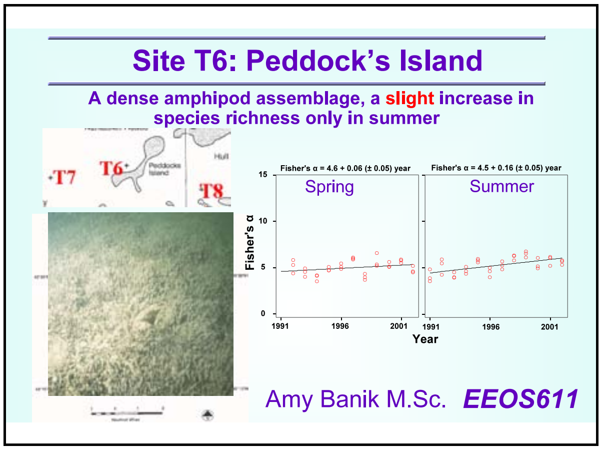
Slide 29 T5A: President's Roads

NOTES:



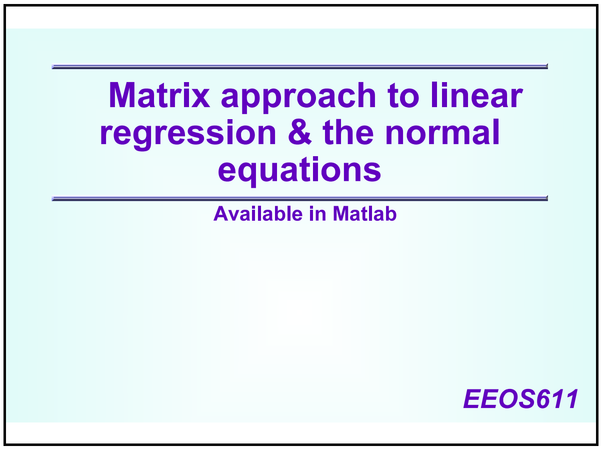
Slide 30 T4: Savin Hill Cove

NOTES:



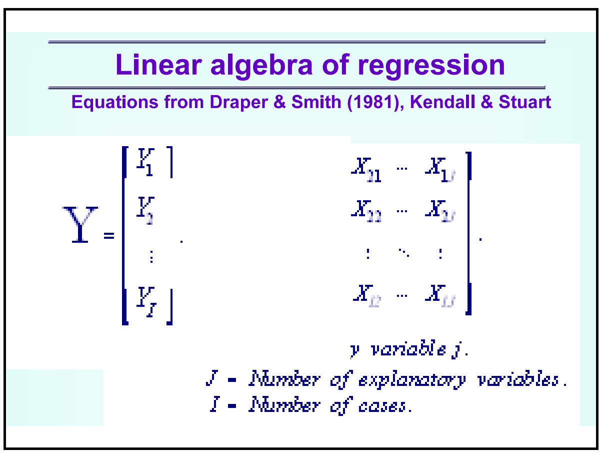
Slide 31 Site T6: Peddock's Island

NOTES:



Slide 32 Matrix approach to linear regression & the normal equations

NOTES:



Slide 33 Linear algebra of regression

NOTES:

The Normal Equations & Matlab

$$Y = X\theta + \epsilon,$$
$$E(\epsilon) = 0.$$
$$V(\epsilon) = E(\epsilon\epsilon') = \sigma^2 I.$$

S=scalar sum of squares.
LS method requires minimization of scalar sum of squares, S.

$$S = (y - X\theta)'(y - X\theta)$$

To minimize S, set $\frac{\partial S}{\partial \theta} = 0.$

Differentiating, $2X'(y - X\theta) = 0.$

$$\hat{\theta} = (X'X)^{-1}X'y.$$

$$\hat{B} = (X'X)^{-1}X'Y \text{ or } \hat{B} = X\backslash Y.$$

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Slide 34 The Normal Equations & Matlab

NOTES:

```
b=X\Y;
Yest=X*b;
resid=Y-Yest;
n=length(Y);
SSb1b0=b'*X'*Y-Y'*ones(n,1)*ones(n,1)'*Y/n;
% Here is the full ANOVA table from Draper & Smith (p. 85) with d.f.
RegSS=b'*X'*Y; ResSS=Y'*Y-b'*X'*Y; TotSS=Y'*Y;
Meanb0=n*(mean(Y)^2);
TotcSS=TotSS-Meanb0;
RF=SSb1b0/(ResSS/(n-length(b))); % The F statistic for the regression:
PRF=fprob(1,n-length(b),RF);
SS=[SSb1b0 1 Ssb1b0 ResSS n-length(b) ResSS/(n-length(b)) TotcSS n-1
TotcSS/(n-1)];
F=[SS(1,3) SS(2,3) SS(1,2) SS(2,2) RF PRF];
s2=SS(2,3);
R2=(b'*X'*Y-Meanb0)/(Y'*Y-Meanb0); % This is R^2
Vb=inv(X'*X)*s2;                    % Equation 2.3.2 in Draper & Smith
%  calculation of the variance of Yest (Draper & Smith, p. 28 & 83)
VYest=diag(Xo'*inv(X'*X)*Xo*s2);
```

Slide 35 Matlab least squares regression

NOTES:

Excursis: Rule of the Bulge vs. Non-linear modeling; from Singer & Willett (2003) Applied longitudinal data analysis.

Non-linear & Weighted regression (will be covered in more detail at the end of Sleuth Chapter 11)

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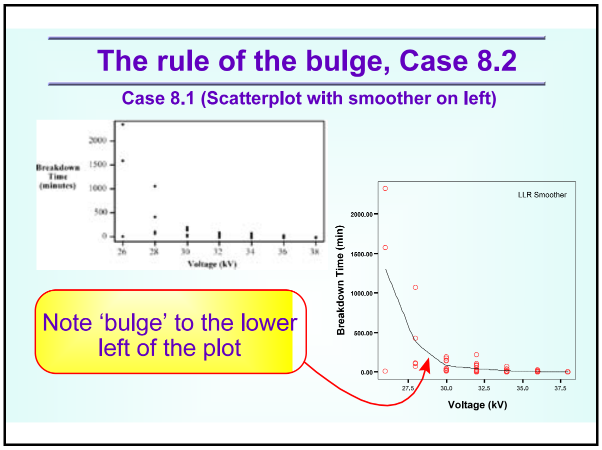
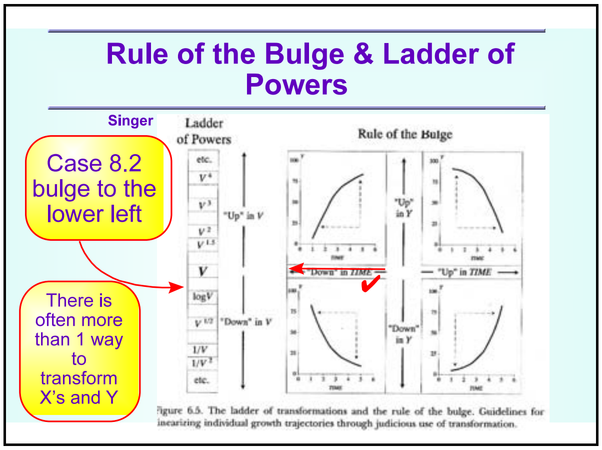
Slide 36 Excursis: Rule of the Bulge vs. Non-linear modeling; from Singer & Willett (2003) Applied longitudinal data analysis.

NOTES:



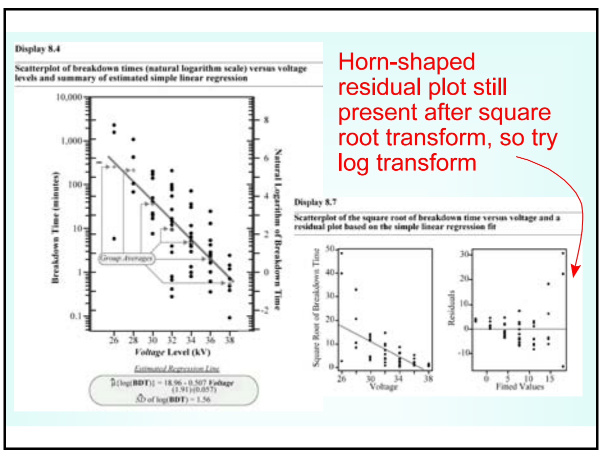
Slide 37 The rule of the bulge, Case 8.2

NOTES:



Slide 38 Rule of the Bulge & Ladder of Powers

NOTES:



Slide 39

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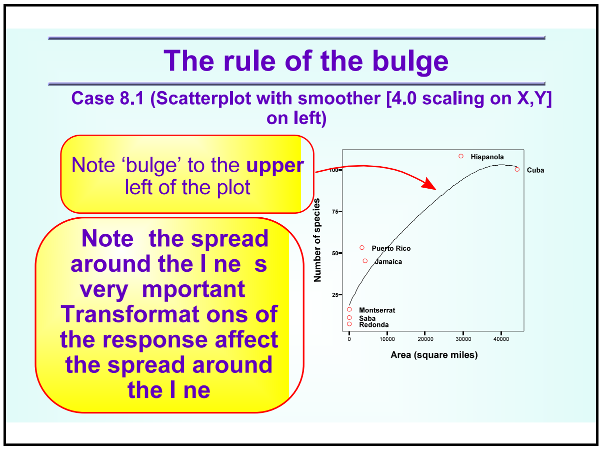
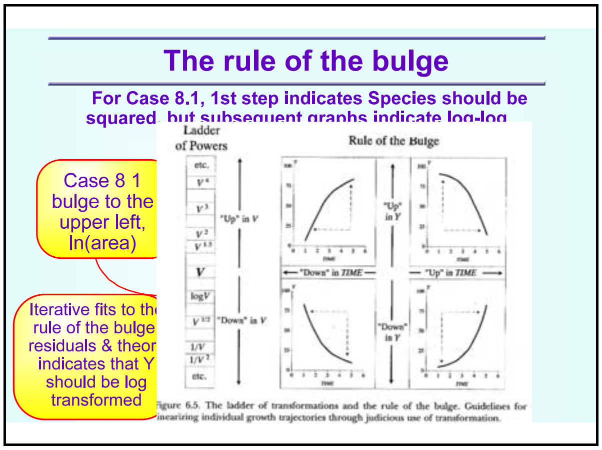


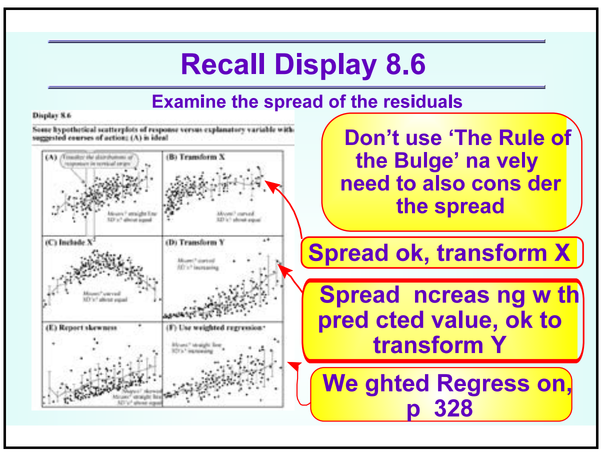
Slide 40 The rule of the bulge

NOTES:



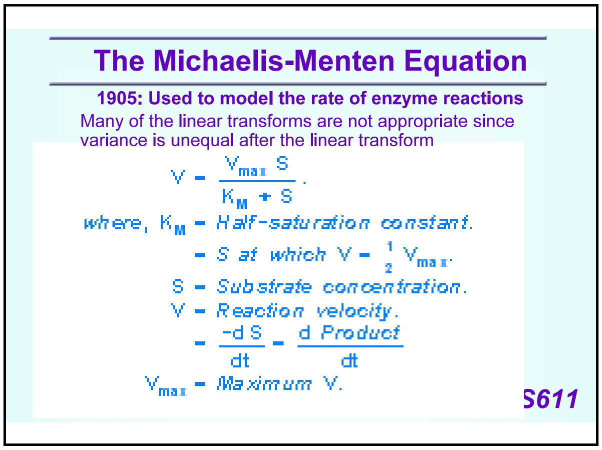
Slide 41 The rule of the bulge

NOTES:



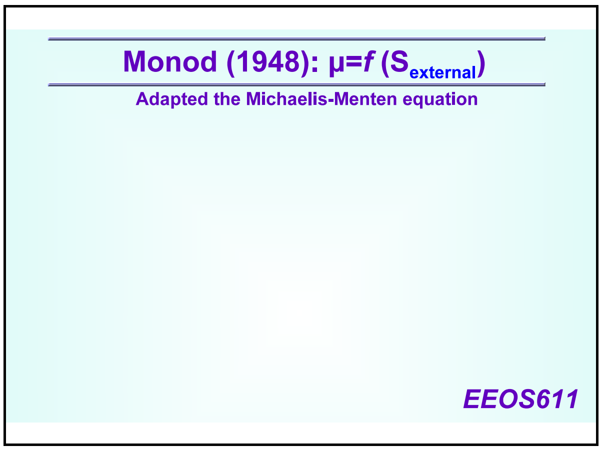
Slide 42 Recall Display 8.6

NOTES:



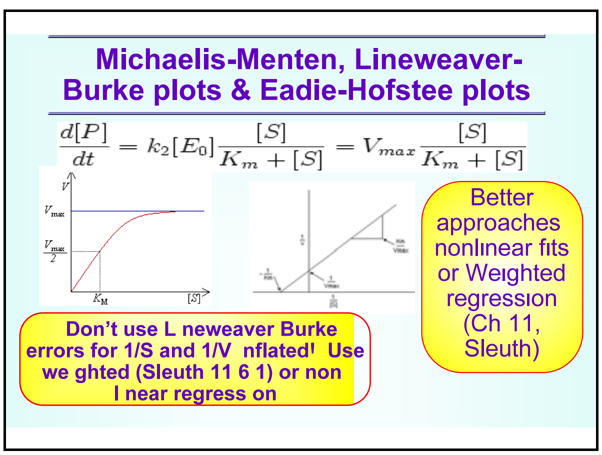
Slide 43 The Michaelis-Menten Equation

NOTES:



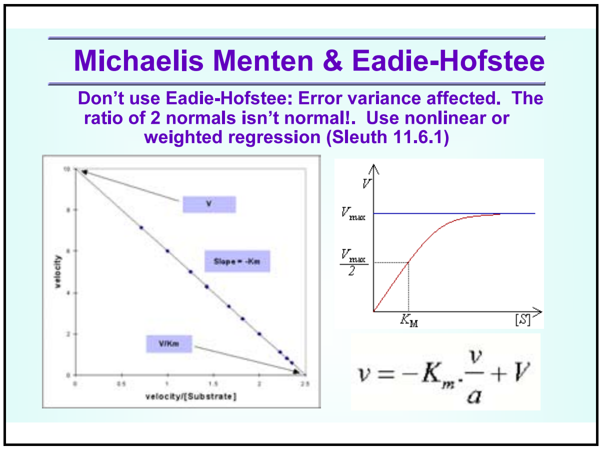
Slide 44 Monod (1948): μ=f (Sexternal)

NOTES:



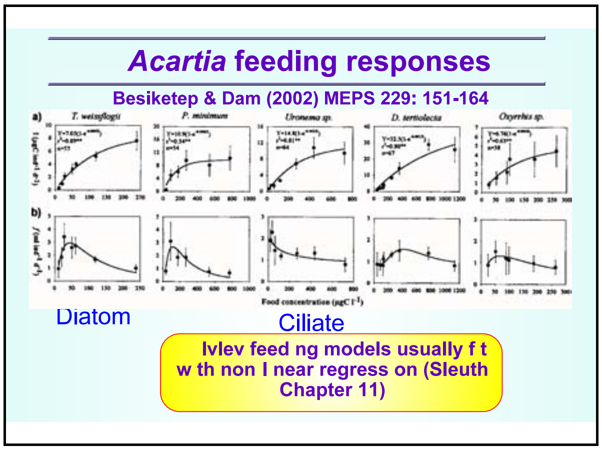
Slide 45 Michaelis-Menten, Lineweaver-Burke plots & Eadie-Hofstee plots

NOTES:



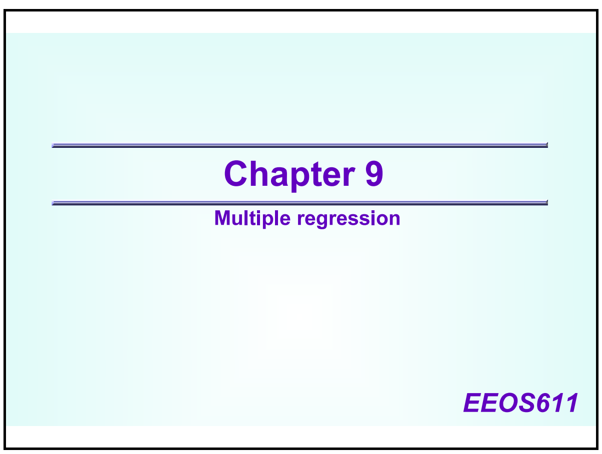
Slide 46 Michaelis Menten & Eadie-Hofstee

NOTES:



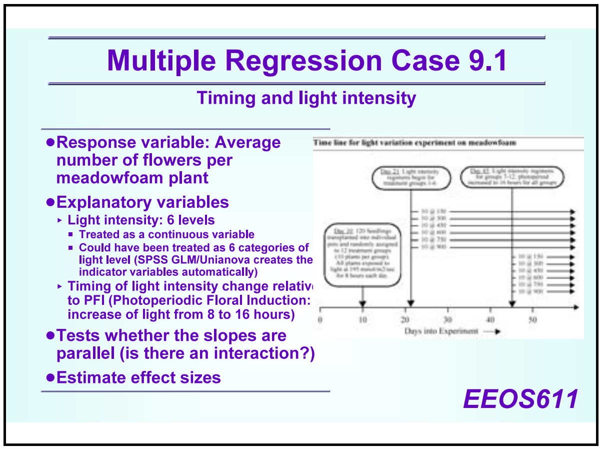
Slide 47 Acartia feeding responses

NOTES:



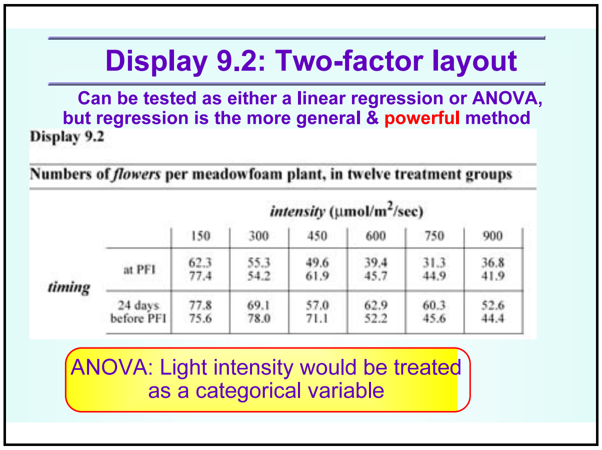
Slide 48 Chapter 9

NOTES:



Slide 49 Multiple Regression Case 9.1

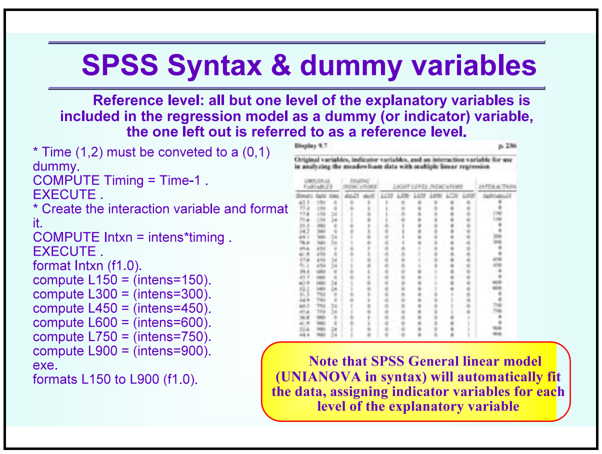
NOTES:



		intensity (µmol/m²/sec)					
		150	300	450	600	750	900
timing	at PFI	62.3 77.4	55.3 54.2	49.6 61.9	39.4 45.7	31.3 44.9	36.8 41.9
	24 days before PFI	77.8 75.6	69.1 78.0	57.0 71.1	62.9 52.2	60.3 45.6	52.6 44.4

Slide 50 Display 9.2: Two-factor layout

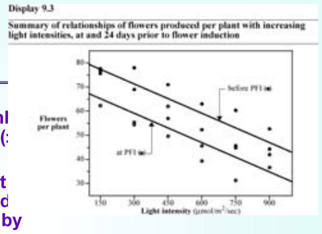
NOTES:



Slide 51 SPSS Syntax & dummy variables

NOTES:

Case 9.1 Statistical summary



- Increasing light intensity decreased the mean numl of flowers per plant by 4 (: plants per 100 μEm^{-2} s^{-1}.
- Beginning light treatment days before PFI increased mean number of flowers by 12.2 (± 5.5) (± ½ 95% CI)

Slide 52 Case 9.1 Statistical summary

NOTES:

Case 9.2: Brain weight

Covariation of Brain weight and demographic traits

- Brain weight and other length measurements are scaled allometrically
  - Y= a Weight^{B}
  - Log-log transforms are the rule for allometric data: log(Y) = log (a) + ß×log (W)
- Is there an association between brain weight (response) and litter size, after controlling for the effect of body weight?
- Is there an association between brain weight (response) and gestation length, after controlling for body weight?

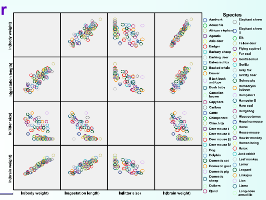
Slide 53 Case 9.2: Brain weight

NOTES:

Case 9.2 Statistical summary

A form of analysis of covariance

- There was convincing evidence that brain weight was associated with either gestation length or litter size after accounting for the effect of body weight (p<0.0001; extra sum of squares F test).
- There was strong evidence that
  - litter size was associated with brain weight after accounting for body weight and gestation (2-sided p value 0.0089) and,
  - Gestation period associated with brain weight after accounting for body weight and litter size (2-sided p value = 0.0038)



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Slide 54 Case 9.2 Statistical summary

NOTES:

Case 9.1, Theory and calculations

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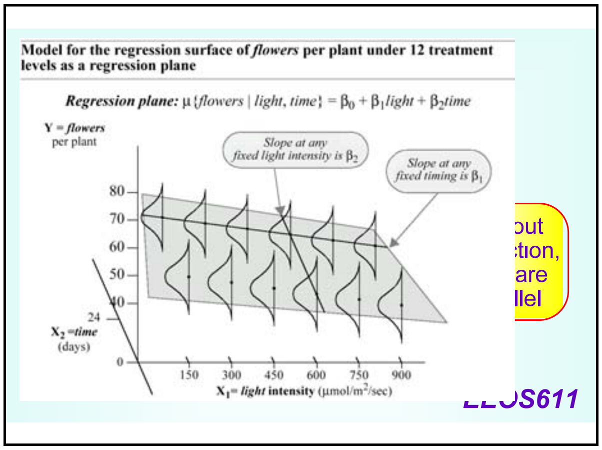
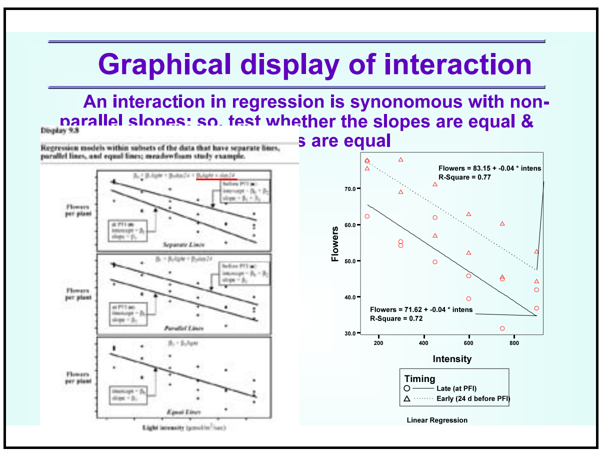
Slide 55 Case 9.1, Theory and calculations

NOTES:



Slide 56 Display 9.5

NOTES:



Slide 57 Graphical display of interaction

NOTES:

SPSS: Solving regression problems

- /analyze/regression
  - Solves all regression problems
  - Multiple models can be set up in advance
  - Indicator (dummy variables) must be computed manually (in syntax or use copy & paste)
- /analyze/general linear model
  - Continuous variables must be entered as covariates; interactions will be calculated for main effects.
  - More tests available, including scatterplots, power analysis, and lack of fit tests
  - With fixed effects, will do analysis as if each level of an explanatory variable was coded as a dummy (=indicator) variable.

Slide 58 SPSS: Solving regression problems

NOTES:

Case 9.2 Allometry of brain size

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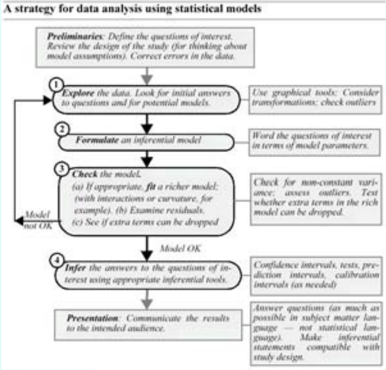
Slide 59 Case 9.2 Allometry of brain size

NOTES:

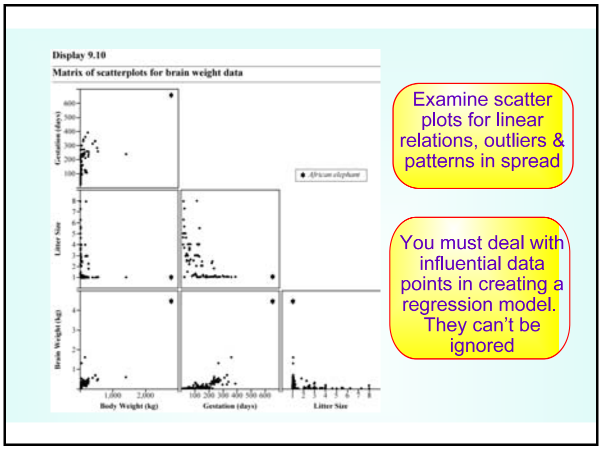
Display 9.9

A strategy for data analysis using statistical models



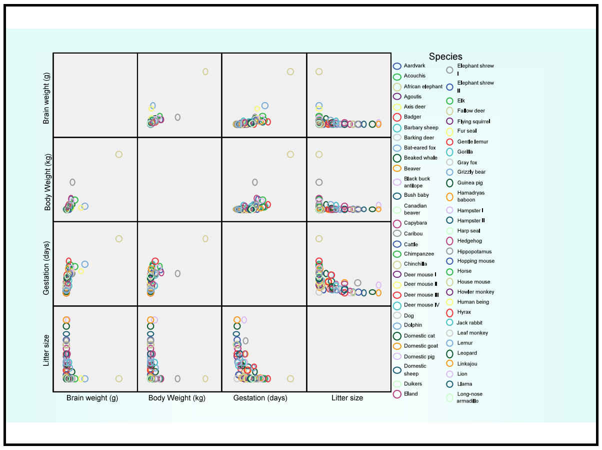
Slide 60 Display 9.9

NOTES:



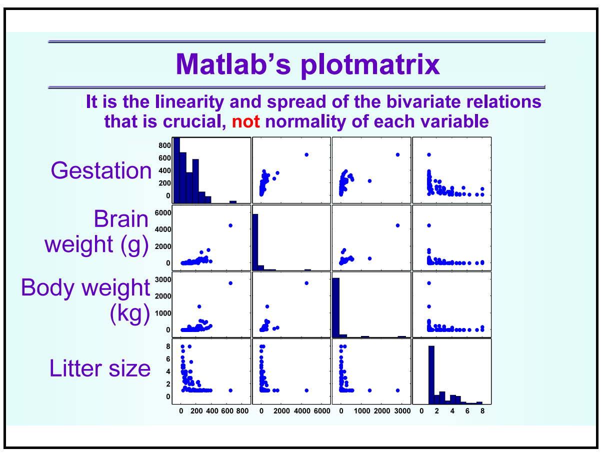
Slide 61 Display 9.10

NOTES:



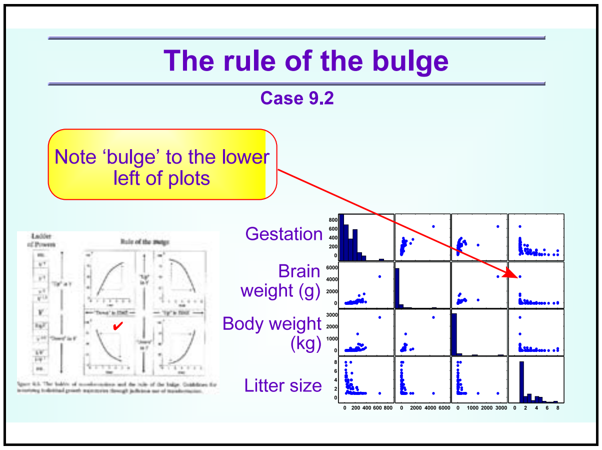
Slide 62

NOTES:



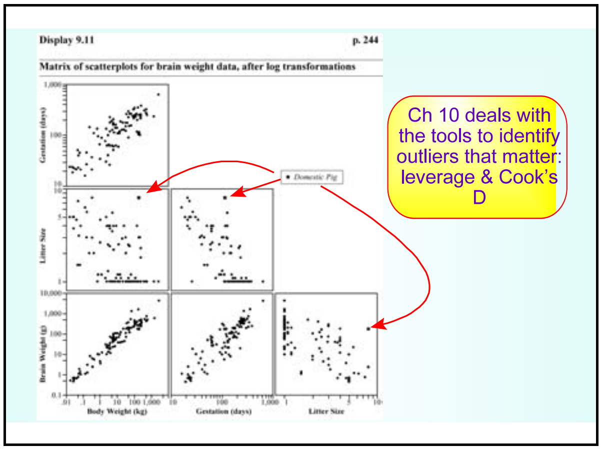
Slide 63 Matlab's plotmatrix

NOTES:



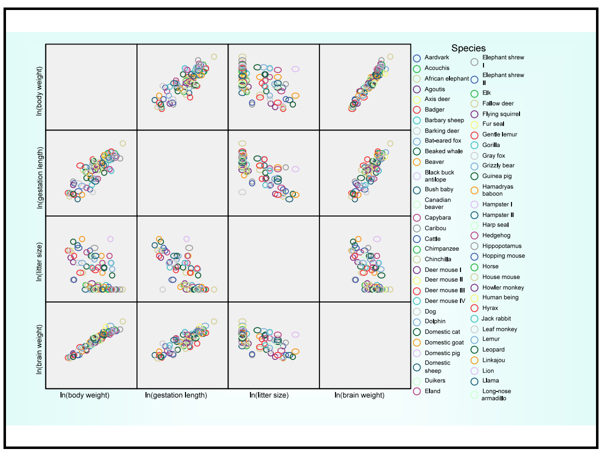
Slide 64 The rule of the bulge

NOTES:



Slide 65 Display 9.11

NOTES:

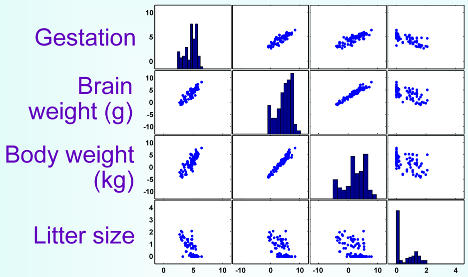


Slide 66

NOTES:

Matlab's plotmatrix.m

It is the linear relation that is important!



Slide 67 Matlab's plotmatrix.m

NOTES:

Case Study 9.2

Testing the effects of Litter size on brain weight, after accounting for body weight and gestation length

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	2.332	.073		31.843	.0000	2.187	2.478
	ln (body wt [kg])	.719	.020	.964	35.300	.0000	.679	.760
2	(Constant)	-.457	.458		-.997	.3212	-1.368	.453
	ln (body wt [kg])	.551	.032	.739	17.033	.0000	.487	.615
	ln (gestation period)	.668	.109	.266	6.141	.0000	.452	.884
3	(Constant)	.855	.662		1.292	.1996	-.459	2.169
	ln (body wt [kg])	.575	.033	.771	17.647	.0000	.510	.640
	ln (gestation period)	.418	.141	.167	2.969	.0038	.138	.698
	ln (litter size)	-.310	.116	-.096	-2.675	.0089	-.540	-.080

a. Dependent Variable: ln (brain wt [g])

Model Summary[d]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics: R Square Change	F Change	df1	df2	Sig. F Change
1	.964[a]	.930	.929	.57807	.930	1246.104	1	94	.0000
2	.975[b]	.950	.949	.49021	.020	37.713	1	93	.0000
3	.977[c]	.954	.952	.47475	.004	7.154	1	92	.0089

a. Predictors: (Constant), ln (body wt [kg])
b. Predictors: (Constant), ln (body wt [kg]), ln (gestation period)
c. Predictors: (Constant), ln (body wt [kg]), ln (gestation period), ln (litter size)
d. Dependent Variable: ln (brain wt [g])

SPSS regression allows multiple models to be fitted sequentially

Slide 68 Case Study 9.2

NOTES:

SPSS syntax Case 9.2

```
REGRESSION
/MISSING LISTWISE
/STATISTICS COEFF CI R ANOVA CHANGE
/CRITERIA=PIN(.05) POUT(.10)
/NOORIGIN
/DEPENDENT lnbrain
/METHOD=ENTER lnbody
/METHOD=ENTER lngest lnbody
/METHOD=ENTER lngest lnbody lnlitt
/SCATTERPLOT=(*ZRESID ,*ZPRED )
/SAVE PRED COOK RESID .
```

Slide 69 SPSS syntax Case 9.2

NOTES:

Case Study 9.2

Testing the effects of gestation length on brain weight, after accounting for body weight and litter size

Coefficients[a]

Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	Upper Bound
1	(Constant)	2.332	.073		31.843	3.8E-052	2.187	2.478
	ln(body weight)	.719	.020	.964	35.300	4.9E-056	.679	.760
2	(Constant)	2.798	.100		27.945	5.0E-047	2.599	2.997
	ln(body weight)	.652	.021	.874	31.339	3.2E-051	.610	.693
	ln(litter size)	-.538	.090	-.166	-5.963	4.4E-008	-.718	-.359
3	(Constant)	.855	.662		1.292	.200	-.459	2.169
	ln(body weight)	.575	.033	.771	17.647	2.8E-031	.510	.640
	ln(litter size)	-.310	.116	-.096	-2.675	.009	-.540	-.080
	ln(gestation length)	.418	.141	.167	2.969	d.0038	.138	.698

a. Dependent Variable: ln(brain weight)

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics: R Square Change	F Change	df1	df2	Sig. F Change
1	.964[a]	.930	.929	.57807	.930	1246.104	1	94	.0000
2	.974[b]	.949	.948	.49430	.019	35.562	1	93	.0000
3	.977[c]	.954	.952	.47475	.004	8.813	1	92	.0038

a. Predictors: (Constant), ln (body wt [kg])
b. Predictors: (Constant), ln (body wt [kg]), ln (litter size)
c. Predictors: (Constant), ln (body wt [kg]), ln (litter size), ln (gestation period)
d. Dependent Variable: ln (brain wt [g])

Slide 70 Case Study 9.2

NOTES:

Extra Sum of Squares F test

= t test for 1 term, but F test can be used for 1 or several terms

10.3.2 F-Test for the Joint Significance of Several Terms

$$\text{F-statistic} = \frac{\left[\frac{\text{Extra sum of squares}}{\text{Number of betas being tested}}\right]}{\text{Estimate of } \sigma^2 \text{ from full model}}$$

ANOVA[d]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	416.400	1	416.400	1246.104	.000[a]
	Residual	31.411	94	.334		
	Total	447.812	95			
2	Regression	425.089	2	212.545	869.915	.000[b]
	Residual	22.723	93	.244		
	Total	447.812	95			
3	Regression	427.076	3	142.359	631.604	.000[c]
	Residual	20.736	92	.225		
	Total	447.812	95			

a. Predictors: (Constant), ln(body weight)
b. **Predictors: (Constant), ln(body weight), ln(litter size)**
c. Predictors: (Constant), ln(body weight), ln(litter size), **ln(gestation length)**
d. Dependent Variable: ln(brain weight)

[(22.723-20.736)/1]/.225
ans = 8.8311
2.97=sqrt(8.8311)

EEOS611

Slide 71 Extra Sum of Squares F test

NOTES:

Comparing Full & Reduced models

Equivalence of *t* tests and F tests for 1 parameter

ANOVA[d]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	416.400	1	416.400	1246.104	.000[a]
	Residual	31.411	94	.334		
	Total	447.812	95			
2	Regression	425.089	2	212.545	869.915	.000[b]
	Residual	22.723	93	.244		
	Total	447.812	95			
3	Regression	427.076	3	142.359	631.604	.000[c]
	Residual	20.736	92	.225		
	Total	447.812	95			

a. Predictors: (Constant), ln(body weight)
b. **Predictors: (Constant), ln(body weight), ln(litter size)**
c. Predictors: (Constant), ln(body weight), ln(litter size), **ln(gestation length)**
d. Dependent Variable: ln(brain weight)

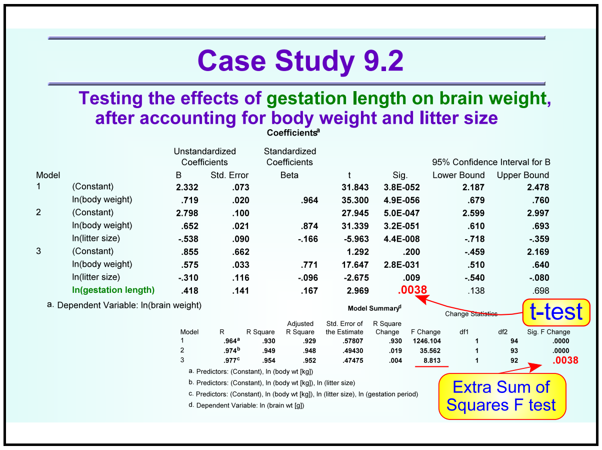
[(22.723-20.736)/1]/.225
ans = 8.8311
2.97=sqrt(8.8311)

Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	Upper Bound
1	(Constant)	2.33	.07		31.8	.000	2.19	2.48
	ln(body weight)	.72	.02	.96	35.3	.000	.68	.76
2	(Constant)	2.80	.10		27.9	.000	2.60	3.00
	ln(body weight)	.65	.02	.87	31.3	.000	.61	.69
	ln(litter size)	-.54	.09	-.17	-6.0	.000	-.72	-.36
3	(Constant)	.85	.66		1.3	.200	-.46	2.17
	ln(body weight)	.58	.03	.77	17.6	.000	.51	.64
	ln(litter size)	-.31	.12	-.10	-2.7	.009	-.54	-.08
	ln(gestation length)	.42	.14	.17	2.97	.004	.14	.70

a. Dependent Variable: ln(brain weight)

Slide 72 Comparing Full & Reduced models

NOTES:



Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95% Confidence Interval for B Lower Bound	95% Confidence Interval for B Upper Bound
1	(Constant)	2.332	.073		31.843	3.8E-052	2.187	2.478
	ln(body weight)	.719	.020	.964	35.300	4.9E-056	.679	.760
2	(Constant)	2.798	.100		27.945	5.0E-047	2.599	2.997
	ln(body weight)	.652	.021	.874	31.339	3.2E-051	.610	.693
	ln(litter size)	-.538	.090	-.166	-5.963	4.4E-008	-.718	-.359
3	(Constant)	.855	.662		1.292	.200	-.459	2.169
	ln(body weight)	.575	.033	.771	17.647	2.8E-031	.510	.640
	ln(litter size)	-.310	.116	-.096	-2.675	.009	-.540	-.080
	ln(gestation length)	.418	.141	.167	2.969	.0038	.138	.698

a. Dependent Variable: ln(brain weight)

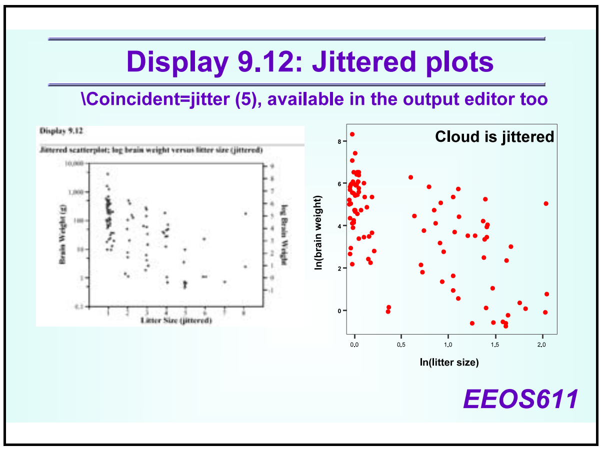
Model Summary[d]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics: R Square Change	F Change	df1	df2	Sig. F Change
1	.964[a]	.930	.929	.57807	.930	1246.104	1	94	.0000
2	.974[b]	.949	.948	.49430	.019	35.562	1	93	.0000
3	.977[c]	.954	.952	.47475	.004	8.813	1	92	.0038

a. Predictors: (Constant), ln (body wt [kg])
b. Predictors: (Constant), ln (body wt [kg]), ln (litter size)
c. Predictors: (Constant), ln (body wt [kg]), ln (litter size), ln (gestation period)
d. Dependent Variable: ln (brain wt [g])

Slide 73 Case Study 9.2

NOTES:



Slide 74 Display 9.12: Jittered plots

NOTES: