

Chapter 14:
Nested & Split-plot Designs
Class 24, 5/6/09 W

Slide 1 Chapter 14:

Nested & Split-plot Designs

NOTES:

HW 16 due Tues 5/12/09 Noon

Submit as Myname-HW16.doc (or *.rtf)

- Read Chapter 14 Multifactor studies without replication
- For Weds read Chapter 23: Elements of Research Design
- For Monday Chapters 18-19: Comparisons of Proportions or Odds
- Final Class: Weds May 13 Research designs Designs
- Class schedule May 6 (Nesting and Experimental Designs), May 11 (Overview of generalized linear models) Exptl design May 13 W Last class
- Wimba Sessions: new times: Monday night 8 pm-9
- Homework 16: Due Tuesday 5/12/09 Noon
- Final Exam 5/22/09 Friday 8-11 am. This is the official time
• Or 5/19/09 Tuesday 8-11 am. I'll find a room

Slide 2 HW 16 due Tues 5/12/09 Noon

NOTES:

Display 23.4

Checklist of tasks involved in the design of a study

- 1. State the objective. *What is the question of interest?*
- 2. Determine the scope of inference.
Will this be a randomized experiment or an observational study?
What experimental or sampling units will be used?
What are the populations of interest?
- 3. Understand the system under study.
- 4. Decide how to measure a response.
- 5. List factors that can affect the response.
Design factors
Factors to vary (treatments & controls)
Factors to fix
Confounding factors
Factors to control by design (blocking)
Factors to control by analysis (covariates)
Factors to control by randomization
- 6. Plan the conduct of the experiment (time line).
- 7. Outline the statistical analysis.
- 8. Determine the sample size ← Attempt this

last ork (16), is due ay 5/12 moved 5/11)

Slide 3

NOTES:

Nested (=hierarchical) ANOVA

A) Testing the Chimp Gender Effect
 B) Testing abundances on the Skagit flats
 C) Testing the Spock Judge Effect (Case 5.2)
 D) Testing airplane training facilities


Slide 4 Nested (=hierarchical) ANOVA

NOTES:

Pseudoreplication= model misspecification

Pseudoreplication: tests using an inappropriate error MS

- 8 buckets enclosing areas of the Skagit intertidal zone
- 4 treatments (2 buckets per treatment)
 - Ambient (only buckets)
 - 50 *Eogammarus*
 - 25 *Crangon*
 - 300 *Eogammarus*
- 8 0.9-cm² cores per bucket after 3 days, 64 total samples
- Is there a treatment effect:
 - Did predators reduce oligochaete abundance?



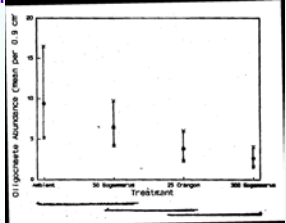
Slide 5 Pseudoreplication= model misspecification

NOTES:

Nested design (Experimental units [buckets] nested within treatment)

Can't be handled as a simple One-way ANOVA

- 8 buckets enclosing areas of the sandy intertidal on the Skagit flats
- 4 treatments
 - Ambient (bucket, no predators)
 - 50 *Eogammarus*
 - 25 *Crangon*
 - 300 *Eogammarus*
- 8 0.9-cm² cores per bucket after 3 days
- Is there a treatment effect:
 - Did predators reduce oligochaete abundance?



Slide 6 Nested design (Experimental units [buckets] nested within treatment)

NOTES:

Nested (hierarchical) ANOVA

NESTED ANOVA of Oligochaeta data:

Main-nested ANOVA

Source	SS	d.f.	MS	F
Total	8.2	63		
Treatment	1.9	3	.65	6.2
Error	6.3	60	.10	

$P(F_{3,60} \geq 6.2) < 0.001$

Source **SS** **d.f.** **MS** **F**

Total 8.2 63

Buckets 4.2 7

Treatment 1.9 3 .65 1.1

Buckets within Treatment 2.3 4 .57 7.9

Error 4.0 56 .07

$P(F_{3,56} \geq 7.9) < 0.001$
 $P(F_{3,4} \geq 1.1) > 0.2$

Slide 7 Nested (hierarchical) ANOVA

NOTES:

Nested ANOVA, with Experimental (or Survey) Units random

Testing the predator effect given bucket-to-bucket variance

Table 9.3. Nested analysis of variance of an experiment with a treatments (1...a) replicated in b units in each treatment (1...j...b in each j) and each sampled with n replicates (1...k...n in each j(i))

Source of variation	Sum of squares	Degrees of freedom	Mean square	Mean square estimates
Among treatments = A	$\sum_{j=1}^a \sum_{i=1}^b (\bar{X}_{.j} - \bar{X})^2 = SS_A$	$a - 1$	$MS_A = SS_A / (a - 1)$	$\sigma_e^2 + n\sigma_{B(A)}^2 + \frac{bn \sum_{j=1}^a (\bar{X}_j - \bar{X})^2}{(a - 1)}$
Among units within each treatment = B(A)	$\sum_{j=1}^a \sum_{i=1}^b (\bar{X}_{.ji} - \bar{X}_{.j})^2 = SS_{B(A)}$	$a(b - 1)$	$MS_{B(A)} = SS_{B(A)} / a(b - 1)$	$\sigma_e^2 + n\sigma_{B(A)}^2$
Within samples	$\sum_{j=1}^a \sum_{i=1}^b \sum_{k=1}^n (X_{ijk} - \bar{X}_{.ji})^2 = SS_w$	$ab(n - 1)$	$MS_w = SS_w / ab(n - 1)$	σ_e^2
Total	$\sum_{j=1}^a \sum_{i=1}^b \sum_{k=1}^n (X_{ijk} - \bar{X})^2 = SS_T$	$abn - 1$		

From Underwood

F = Among Treatment MS /
 [Among Experimental Units within Treatment MS]

Slide 8 Nested ANOVA, with Experimental (or Survey) Units random

NOTES:

How to perform Nested ANOVA

1 of 3, 3 methods if experimental units are random factors

- Decide *a priori* whether experimental or survey units (Rocky areas, companies, chimps, buckets, classes) are random or fixed factors
 - If random, the design is a mixed model ANOVA with treatment as fixed and experimental (observational) units as random
 - If fixed, the design is a fixed-factor nested ANOVA
 - The results are identical to that obtained using linear contrasts
 - Different denominator mean squares for testing main effects with random and fixed 'units'

Slide 9 How to perform Nested ANOVA

NOTES:

How to perform a Nested ANOVA

2 of 3

- Method 1: A tried-and-true method: perform analyses as separate one-way ANOVAs
 - 1) Test for differences among experimental units to produce 'among Experimental Units' SS [and df] (differences among the 8 buckets in the Skagit example)
 - 2) Combine experimental units within treatments and perform 1-way ANOVA to produce among treatment SS [and df]
 - 3) subtract Among Treatment SS from 'Among experimental units SS' to produce 'Units within treatment' SS [and df]
 - 4) Calculate Mean squares by dividing by df
 - 5) Test treatment effects
 - A) Mixed model (Units as random factors): F Test Among treatment MS/Units within Treatment' MS
 - B) Fixed model: Test Among treatment MS/ Error MS'

Slide 10 How to perform a Nested ANOVA

NOTES:

How to perform a Nested ANOVA

3 of 3

- Method 2: Alternatively for mixed model, calculate the mean response within each experimental unit and perform the 1-way ANOVA on the means
 - Hurlbert (1984) noted that nested ANOVAs are identical to performing ANOVAs on the unit means
- Method 3: SPSS. Must use syntax to specify the nested or hierarchical ANOVA tests.

```
* Case 1401.
UNIANOVA
  Inmin BY sign chimp sex
  /METHOD = SSTYPE(3)
  /INTERCEPT = INCLUDE
  /CRITERIA = ALPHA(.05)
  /RANDOM=chimp
  /DESIGN = sign sex chimp(sex) .
```

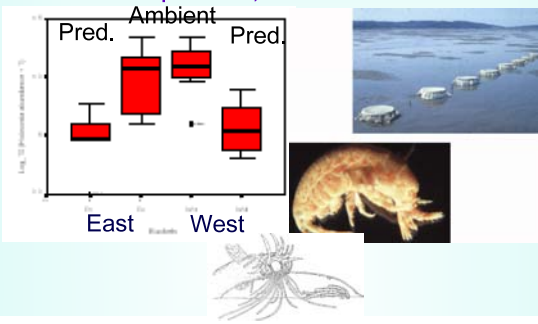


Slide 11 How to perform a Nested ANOVA

NOTES:

Effects of predation on *Hobsonia*

8 cores per bucket, Eastern & Western Blocks



Slide 12 Effects of predation on *Hobsonia*

NOTES:

Slide 13 Nested ANOVA

Nested ANOVA

Pooling buckets within treatment: beware pseudoreplication

Table 1. Nested ANOVA of Log₁₀ transformed abundances.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	Sig.
Total	3.44	31			
Blocks	2.08	7	0.297	11.9	3.3e-05
Treatment (Predator vs. Control)	1.98	1	1.98	66.6	0.015
Blocks within Treatment ^a	0.06	2	0.03	0.2	0.82
Error	1.07	20	0.054		

Pooling experimental units within treatments might be justified in this case, but reviewers will suspect pseudoreplication

ANOVA if Units within treatment pooled with error

Table 1. One-way ANOVA of Log₁₀ transformed abundances.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	Sig.
Total	3.44	31			
Treatment	1.98	1	1.98	35.8	1.5e-06
Error	1.46	30	0.049		

NOTES:

Slide 14 Mixed model nested ANOVA equivalent to ANOVA on the means

Mixed model nested ANOVA equivalent to ANOVA on the means

There is still an advantage to replication: increased precision

Table 1. Means of log₁₀+1 abundances.

Treatment	Block	Bucket	Mean Log ₁₀ transformed Abundance
Predator	East	E1	.49
Ambient	East	E4	.98
Ambient	West	W3	1.07
Predator	West	W4	.57

Table 1. One-way ANOVA of Log₁₀ transformed abundances.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	Sig.
Total	3.44	31			
Blocks	2.08	7	0.297	11.9	3.3e-05
Treatment (Predator vs. Control)	1.98	1	1.98	66.6	0.015
Blocks within Treatment ^a	0.06	2	0.03	0.2	0.82
Error	1.07	20	0.054		

Table 1. One-way ANOVA of 4 means.

	Sum of Squares	df	Mean Square	F	Sig.
Treatment	.248	1	.248	66.6	.015
Error	7.4e-03	2	3.7e-03		
Total	.255	3			

NOTES:

Blocked ANOVA removes East vs. West variance

The Block x Treatment Mean square used to test predator effect; block is regarded as a random factor

Table 1. Blocked ANOVA, based on four means. The main effect (treatment & Block) significance levels are based on the F_{1,1} distribution, with the Block x Treatment mean square in the denominator of the test statistic.

Source of Variation	Sum of Squares	df	Mean Square	F	Sig.
Block (East vs. West)	7.4e-03	1	7.4e-03	289	0.040
Treatment	.248	1	.248	9679	0.010
Block x Treatment (used to estimate error)	2.6e-05	1	2.6e-05		
Total	.255	3	8.5e-02		

Even though the F statistic denominator is based on 1 df, the blocking improved power



Slide 15 Blocked ANOVA removes East vs. West variance

NOTES:

Nesting vs. Blocked ANOVA

- Nesting: subsamples from experimental units can **not** be treated as replicates (pooling may be permissible only if "Exptl Units within Treatment MS has $p > 0.5$, see Winer et al.)
- In designing an experiment or survey with a nested structure, investigator should strive to replicate experimental or survey units, **not** subsamples of experimental or survey units
- The final df will be partitioned as if only the means for each experimental unit were analyzed
 - There is a huge benefit from taking replicate subsamples in that the estimated means will be less variable (Avg/\sqrt{n} , central limit theorem)
- Blocking usually produces a more powerful design

Slide 16 Nesting vs. Blocked ANOVA

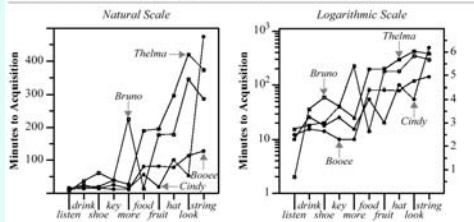
NOTES:

Is there a gender difference in learning?

4 experimental units (chimps), 2 genders

Display 14.10

Chimpanzee data plots: natural scale and logarithmic scale



Slide 17 Is there a gender difference in learning?

NOTES:

Chimp Nested ANOVA

Sex is a 0,1 indicator variable
Chimp is the experimental unit, chimp variability nested within gender

UNIANOVA

Inmin BY sign chimp sex

/METHOD = SSTYPE(3)

/INTERCEPT = INCLUDE

/CRITERIA = ALPHA(.05)

/RANDOM=chimp

/DESIGN = sign sex *chimp* within sex.

* Or,
UNIANOVA
Inmin BY sign chimp sex
/METHOD = SSTYPE(3)
/INTERCEPT = INCLUDE
/CRITERIA = ALPHA(.05)
/RANDOM=chimp
/DESIGN = sign sex *chimp*(sex) .

The two italicized syntax lines specify identical nested analyses.

Slide 18 Chimp Nested ANOVA

NOTES:

If chimps random, test gender effect over 'among chimp within gender' mean square

If chimps are a random sample of chimps within gender, no effect of gender ($p=0.82$) on time to learn signs

Tests of Between-Subjects Effects

Dependent Variable: In(Minutes)

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	Hypothesis	615.158	1	615.158	238.173	.004
	Error	5.166	2	2.583 ^a		
sign	Hypothesis	45.690	9	5.077	7.765	1.50E-005
	Error	17.653	27	.654 ^b		
sex	Hypothesis	.167	1	.167	.065	.823
	Error	5.166	2	2.583 ^a		
chimp(sex)	Hypothesis	5.166	2	2.583	3.950	.031
	Error	17.653	27	.654 ^b		

a. MS(chimp(sex))
b. MS(Error)

Note, there is evidence for significant chimp-within-gender variance ($p=0.031$)

Slide 19 If chimps random, test gender effect over 'among chimp within gender' mean square

NOTES:

Chimps should be regarded as a fixed not a random factor

Test treatment over error mean square, see Neter et al. (1996)
Note that nested ANOVAs are usually handled as mixed models with experimental or survey units being random not fixed effects. These 4 chimps could not be regarded as random or even representative samples of a larger chimp population, so chimp within gender variability should be a fixed factor and inferences to a larger chimp population are not warranted.

Tests of Between-Subjects Effects

Dependent Variable: In(Minutes)

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model		51.023 ^a	12	4.252	6.50	2.8E-005
Intercept		615.158	1	615.158	940.90	1.6E-022
sign		45.690	9	5.077	7.76	1.5E-005
sex		.167	1	.167	.26	.617
chimp(sex)		5.166	2	2.583	3.95	.031
Error		17.653	27	.654		
Total		683.834	40			
Corrected Total		68.675	39			

a. R Squared = .743 (Adjusted R Squared = .629)

No gender effect

Slide 20 Chimps should be regarded as a fixed not a random factor

NOTES:

Nested ANOVA with chimp as a fixed effect is identical to a linear contrast on sex (specified a priori)

- UNIANOVA
- Inmin BY sign chimp
- /METHOD = SSTYPE(3)
- /LMatrix = "Male v. Female " chimp 1/2 1/2 -1/2 -1/2
- /INTERCEPT = INCLUDE
- /CRITERIA = ALPHA(.05)
- /DESIGN = sign chimp.

The denominator for Treatment F test is the error mean square

Slide 21 Nested ANOVA with chimp as a fixed effect is identical to a linear contrast on sex (specified a priori)

NOTES:

Results of testing gender effect using linear contrast: chimp 1/2 1/2 -1/2 -1/2

Contrast Results (K Matrix)

Contrast	Contrast Estimate	Dependent Variable
L1		ln(Minutes)
	Hypothesized Value	-0.13
	Difference (Estimate - Hypothesized)	.00
	Std. Error	-0.13
	Sig.	.26
	95% Confidence Interval for Lower Bound	.62
	Difference Upper Bound	-0.65
		0.40

a. Based on the user-specified contrast coefficients (1/2, 1/2, -1/2, -1/2) for Male and Female

Slide 22 Results of testing gender effect using linear contrast: chimp 1/2 1/2 -1/2 -1/2

NOTES:

Neter et al. On distinguishing between crossed and nested designs

Slide 23 Neter et al. On distinguishing between crossed and nested designs

NOTES:

Nested vs. Crossed Factors

Neter et al.'s (1996) cities and airport mechanic training school example: mechanic training in schools in Atlanta, Chicago & San Francisco (2 instructors per school x 2 classes per instructor)

Score	instructor	city
25	1	Atlanta
29	1	Atlanta
14	2	Atlanta
11	2	Atlanta
11	3	Chicago
6	3	Chicago
22	4	Chicago
19	4	Chicago
17	5	San Francisco
20	5	San Francisco
5	6	San Francisco
2	6	San Francisco

TABLE 28.1 Sample Data for Nested Two-Factor Study—Training School Example (class learning scores, coded).

Factor A (school)	Factor B (instructor)		Average
	1	2	
Atlanta	25 29	14 11	$\bar{Y}_{1.} = 19.75$
Chicago	11 6	22 18	$\bar{Y}_{2.} = 14.25$
San Francisco	17 20	5 2	$\bar{Y}_{3.} = 11.00$
Average	$\bar{Y}_{.1} = 8.5$	$\bar{Y}_{.2} = 3.5$	$\bar{Y}_{..} = 15$

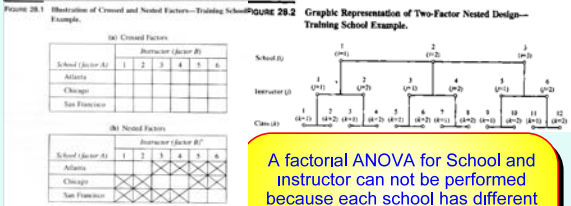
Slide 24 Nested vs. Crossed Factors

NOTES:

Slide 25 Nested vs. Crossed

Nested vs. Crossed

The training example is nested, not crossed
 Instructor-to-instructor variability is nested within cities &
 class-to-class variability is nested within instructors

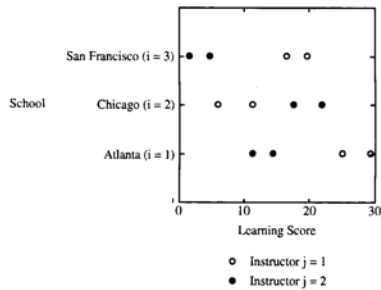


A factorial ANOVA for School and instructor can not be performed because each school has different instructors. Instructors are a nested factor, not crossed.

NOTES:

Slide 26

FIGURE 28.3 Dot Plots of Class Learning Scores—Training School Example.



prior whether instructors are a fixed or random factor

NOTES:

Are experimental units fixed or random factors?

If exptl units are fixed effect, test main effects vs. Error term;
 If random, test main effect vs. 'Experimental units within treatment' mean square

TABLE 28.5 Expected Mean Squares for Nested Balanced Two-Factor Designs with Random Factor Effects (B nested within A).

Mean Square	Expected Mean Square	
	A Fixed, B Random	A Random, B Random
MSA	$\sigma^2 + bn \frac{\sum \alpha_i^2}{a-1} + n\sigma_B^2$	$\sigma^2 + bn\sigma_A^2 + n\sigma_B^2$
MSB(A)	$\sigma^2 + n\sigma_B^2$	$\sigma^2 + n\sigma_B^2$
MSE	σ^2	σ^2

Experimental Units (instructors) within treatment (cities)

Test for	Appropriate Test Statistic	
	A Fixed, B Random	A Random, B Random
Factor A	MSA/MSB(A)	MSA/MSB(A)
Factor B(A)	MSB(A)/MSE	MSB(A)/MSE

Slide 27 Are experimental units fixed or random factors?

NOTES:

Two-factor Fixed Effects model

Neter et al. (1996) consider instructors to be a fixed effect, not a random effect

TABLE 28.3 ANOVA Table for Nested Balanced Two-Factor Fixed Effects Model (28.7) (B nested within A).

Source of Variation	SS	df	MS	F (MS)
Factor A	$SSA = bn \sum (\bar{Y}_{i..} - \bar{Y}_{...})^2$	$a - 1$	MSA	$\sigma^2 + bn \frac{\sum \alpha_i^2}{a - 1}$
Factor B (within A)	$SSB(A) = n \sum \sum (\bar{Y}_{ij.} - \bar{Y}_{i..})^2$	$a(b - 1)$	MSB(A)	$\sigma^2 + n \frac{\sum \sum \beta_{ij}^2}{a(b - 1)}$
Error	$SSE = \sum \sum \sum (Y_{ijk} - \bar{Y}_{ij.})^2$	$ab(n - 1)$	MSE	σ^2
Total	$SSTO = \sum \sum \sum (Y_{ijk} - \bar{Y}_{...})^2$	$abn - 1$		

Test for City effect over error mean square, not over instructor within city mean square

Slide 28 Two-factor Fixed Effects model

NOTES:

Example. Based on the analysis of variance in Table 28.4a for the training school example, we conduct the first test to determine whether or not main school effects exist. The alternatives are given in (28.17a), and test statistic (28.17b) here is:

$$F^* = \frac{78.25}{7.00} = 11.2$$

Strong evidence that the schools differ in learning effects.

For level of significance $\alpha = .05$, we require $F(.95; 2, 6) = 5.14$. Since $F^* = 11.2 > 5.14$, we conclude that the three schools differ in mean learning effects. The P -value of the test is .0094.

Next is a test for differences in mean learning effects between instructors within each school. The alternatives are given in (28.18a), and test statistic (28.18b) here is:

$$F^* = \frac{189.17}{7.00} = 27.0$$

Strong evidence for differences among instructors within schools.

For $\alpha = .05$, we require $F(.95; 3, 6) = 4.76$. Since $F^* = 27.0 > 4.76$, we conclude that instructors within at least one school differ in terms of mean learning effects. The P -value of this test is .0007.

Slide 29

NOTES:

This example: Fixed factor (Model I) nested ANOVA

- Neter et al. (1996) analyze the City and instructor problem as a Model I or fixed effects Nested ANOVA
- If the goal is to assess whether there are differences in the quality of the city training facilities (rather than the instructors), the model should probably be analyzed as a mixed model ANOVA, with instructors as random factors
 - Note that there is no evidence that instructors are a random sample of some larger population of instructors, so the fixed model is appropriate here
 - The inference can **not** be made that the city effect would be observed with a different set of instructors
- With a mixed model nested ANOVA, the city effect is assessed vs. 'instructors within city' MS



Slide 30 This example: Fixed factor (Model I) nested ANOVA

NOTES:

Instructors as fixed factors

UNIANOVA score BY city instructor
 /METHOD = SSTYPE(3)
 /INTERCEPT = INCLUDE
 /CRITERIA = ALPHA(.05)
 /DESIGN = city instructor(city)

Tests of Between-Subjects Effects

* Dependent Variable: Score					
Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	724.800 ^a	5	144.960	20.688	.001
Intercept	2700.000	1	2700.000	385.714	.000
city	156.500	2	78.250	11.179	.009
instructor(city)	567.500	3	189.167	27.024	.001
Error	42.000	6	7.000		
Total	3486.000	12			
Corrected Total	766.800	11			

a. R Squared = .945 (Adjusted R Squared = .899)

Slide 31 Instructors as fixed factors

NOTES:

Instructors as random factors

Permits inferences beyond these 6 instructors, if these instructors can be regarded as random samples from a larger pool of instructors

UNIANOVA score BY city instructor
 /METHOD = SSTYPE(3)
 /INTERCEPT = INCLUDE
 /CRITERIA = ALPHA(.05)
/RANDOM=instructor
 /DESIGN = city instructor(city).

Slide 32 Instructors as random factors

NOTES:

If instructors treated as a random factor, there is no effect of city

Tests of Between-Subjects Effects

Dependent Variable: Score						
Source	Hypothesis	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept		2700.000	1	2700.000	14.273	.032
city	Hypothesis	156.500	2	78.250	.414	.694
	Error	567.500	3	189.167 ^a		
instructor(city)	Hypothesis	567.500	3	189.167	27.024	.001
	Error	42.000	6	7.000 ^b		

a. MS(instructor(city))
 b. MS(Error)

Business implication:
 transfer or train the instructors rather than close down an inferior city's airport repair facility

Slide 33 If instructors treated as a random factor, there is no effect of city

NOTES:

<p style="text-align: center;">Repeated Measures ANOVA</p> <p style="text-align: center;">Chapter 16</p> <p>If the responses are from repeated measurements of the same individual (plot, beaker, animal), the observations can not be treated as if they are independent. Observations from the same individual are often positively correlated. A repeated measures analysis must be used. It is usually advantageous to do so, because more powerful tests of hypotheses are possible.</p>	<p style="text-align: center;">Slide 34 Repeated Measures ANOVA</p> <hr/> <p>NOTES:</p> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>
<p style="text-align: center;">Repeated measures designs</p> <p style="text-align: center;">Sampling units serve as their own controls</p> <ul style="list-style-type: none"> ● Observations are collected at different times on the same sampling unit <ul style="list-style-type: none"> ▸ Placebo, control drug trials ▸ Sampling the same quadrat (area) through time ▸ The errors within a subject are correlated ● Within and between subject factors <ul style="list-style-type: none"> ▸ Within subject factors: two different observations of the same sampling unit (e.g., time or drug dose) ▸ Between subject factors: disjunct groups of sampling units (patients, plots, treatment, order of treatment) ● Repeated measures designs offer much more powerful tests than non-repeated designs 	<p style="text-align: center;">Slide 35 Repeated measures designs</p> <hr/> <p>NOTES:</p> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>
<p style="text-align: center;">Advantages & Disadvantages</p> <p style="text-align: center;">From Neter et al. (1996)</p> <ul style="list-style-type: none"> ● Advantages <ul style="list-style-type: none"> ▸ Increased precision because between-subject variability is excluded from experimental error ▸ Economizes on subjects ▸ When the shape of the time effect is important, measures of the same subject (fitting population growth is possible) ● Disadvantages & Interferences <ul style="list-style-type: none"> ▸ Order effect ▸ Carryover effect (bland soup, good soup; grocery shopping [full & empty trips]) ● Other disadvantages: more complicated assumptions and models 	<p style="text-align: center;">Slide 36 Advantages & Disadvantages</p> <hr/> <p>NOTES:</p> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>

Analyzing repeated measures designs

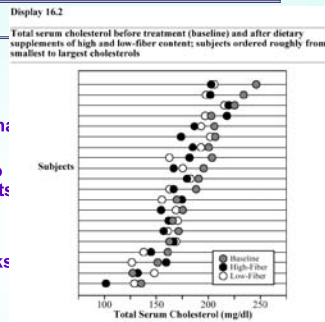
- Three classes of repeated measures analysis**
- Repeated measures designs
 - Common features
 - Repeated measures from the same individual, bottle or area
 - Correlated observations
 - Between-individual heterogeneity
 - Correlations within individuals
 - Measurement error
 - 1) Univariate repeated measures (Winer et al.)
 - Developed by Fisher, individuals as a random effect
 - Assumes Sphericity
 - Equivalent to a split-plot design with subjects as a fixed factor
 - 2) Multivariate Repeated Measures (Profile analysis)
 - Hotelling's T² and other multivariate statistics
 - Scheiner 2001 on MANOVA & von Ende (2001) on repeated measures
 - 3) Longitudinal models
 - Singer & Willett multilevel longitudinal models
 - Multilevel Longitudinal models (Singer & Willett 2003, Fitzmaurice et al. 2004)
 - More detailed analysis of within-individual trajectories and error structure
 - Allows autocorrelated errors in assessing time effects
 - SAS Proc mixed, SPSS Mixed
 - If there are just two paired variables, use a paired *t* test
 - Nonparametric test: Friedman's ANOVA

Slide 37 Analyzing repeated measures designs

NOTES:

Case study 16.2

- 20 subjects
- Randomized, double blind, crossover trial
- 1-wk baseline on norm: diet
- Randomly assigned to High and low fiber diets
- 6 weeks on blood chemistry
- Normal diet for 2 weeks
- Crossed over to other diet.



Slide 38 Case study 16.2

NOTES:

SPSS Syntax for cholesterol

GLM multivariate (=MANOVA) in SPSS advanced modules

* Compare to Display 1610 in text.
 GLM
 baseline hifibr lofibr
 /WSFACTOR = diet 3 Difference
 /METHOD = SSTYPE(3)
 /PLOT = PROFILE(diet)
 /EMMEANS = TABLES(diet)
 /PRINT = DESCRIPTIVE
 /CRITERIA = ALPHA(.05)
 /WSDESIGN = diet .

The assumption for the multivariate approach is that the vector of the response variables follow a multivariate normal distribution, and the variance-covariance matrices are equal across the cells formed by the between-subjects effects.

Slide 39 SPSS Syntax for cholesterol

NOTES:

Large differences in diet effects on cholesterol

No reason to reject the sphericity assumption

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
diet	Sphericity Assumed	2410.300	2	1205.150	9.775	3.759E-004
	Greenhouse-Geisser	2410.300	1.999	1205.714	9.775	3.768E-004
	Huynh-Feldt	2410.300	2.000	1205.150	9.775	3.759E-004
	Lower-bound	2410.300	1.000	2410.300	9.775	5.557E-003
Error(diet)	Sphericity Assumed	4685.033	38	123.290		
	Greenhouse-Geisser	4685.033	37.982	123.348		
	Huynh-Feldt	4685.033	38.000	123.290		
	Lower-bound	4685.033	19.000	246.581		

Report that, ' univariate repeated measures ANOVA' found strong evidence for diet effects on cholesterol (p=0.0004)

Slide 40 Large differences in diet effects on cholesterol

NOTES:

Effect of high fiber diet

Little difference between High and Low-fiber diet
Note Bonferroni CI's narrower than the [-8.8 10.1] CI's using the T² multiplier (Display 16.10)

Pairwise Comparisons

Measure: MEASURE_1

(I) diet	(J) diet	Mean Difference (I-J)	Std. Error	Sig. ^a	95% Confidence Interval for Difference ^a	
					Lower Bound	Upper Bound
1	2	13.850*	3.533	.003	4.576	23.124
	3	13.000*	3.473	.004	3.882	22.118
2	1	-13.850*	3.533	.003	-23.124	-4.576
	3	-.850	3.528	1.000	-10.110	8.410
3	1	-13.000*	3.473	.004	-22.118	-3.882
	2	.850	3.528	1.000	-8.410	10.110

Based on estimated marginal means
*. The mean difference is significant at the .05 level.
a. Adjustment for multiple comparisons: Bonferroni.

Slide 41 Effect of high fiber diet

NOTES:

Display 16.11

High-baseline and high-low cholesterol levels, by the order of assignment

Slide 42

NOTES:

Test for an order effect

Order is a between-subjects factor, diet is within subjects
* Repeated measures between subjects test.

GLM
baseline hifibr lofibr **BY order**
/WSFACTOR = diet 3 Difference
/CONTRAST (order)=Difference
/METHOD = SSTYPE(3)
/PLOT = PROFILE(diet order*diet)
/CRITERIA = ALPHA(.05)
/WSDESIGN = diet
/DESIGN = order .

Slide 43 Test for an order effect

NOTES:

No order effect on diet

Sphericity assumption appears justified here.

Measure: MEASURE_1		Tests of Within-Subjects Effects				
Source		Type III Sum of Squares	df	Mean Square	F	Sig.
diet	Sphericity Assumed	2410.300	2	1205.150	10.017	.00035
	Greenhouse-Geisser	2410.300	1.993	1209.302	10.017	.00035
	Huynh-Feldt	2410.300	2.000	1205.150	10.017	.00035
	Lower-bound	2410.300	1.000	2410.300	10.017	.00536
	Upper-bound	2410.300	1.000	2410.300	10.017	.00536
diet * order	Sphericity Assumed	354.033	2	177.017	1.471	.243
	Greenhouse-Geisser	354.033	1.993	177.826	1.471	.243
	Huynh-Feldt	354.033	2.000	177.017	1.471	.243
	Lower-bound	354.033	1.000	354.033	1.471	.241
	Upper-bound	354.033	1.000	354.033	1.471	.241
Error(diet)	Sphericity Assumed	4331.000	36	120.306		
	Greenhouse-Geisser	4331.000	35.876	120.720		
	Huynh-Feldt	4331.000	36.000	120.306		
	Lower-bound	4331.000	18.000	240.611		
	Upper-bound	4331.000	18.000	240.611		

There is little evidence (repeated measures ANOVA diet*order effect, p=0.24) that diet order affects cholesterol differences among diets

Slide 44 No order effect on diet

NOTES:

Friedman's Nonparametric repeated measures ANOVA

Example from Hollander & Wolfe

FIG. 1.1. A DISTRIBUTION-FREE TEST FOR GENERAL ALTERNATIVES. 275

FIG. 1.1. These methods of connecting first base: ○ path = round out method, + path = narrow angle method, solid path = wide angle method.

Slide 45 Friedman's Nonparametric repeated measures ANOVA

NOTES:

Rank within subject

Must assume no Player x Treatment Interaction

TABLE 7.1. Rounding First Race Times

Players	Methods		
	Round Out	Narrow Angle	Wide Angle
1	5.40(1)	5.50(2)	5.55(3)
2	5.85(3)	5.70(1)	5.75(2)
3	5.20(1)	5.60(3)	5.50(2)
4	5.55(3)	5.50(2)	5.40(1)
5	5.90(3)	5.85(2)	5.70(1)
6	5.45(1)	5.55(2)	5.60(3)
7	5.40(2.5)	5.40(2.5)	5.35(1)
8	5.45(2)	5.50(3)	5.35(1)
9	5.25(1)	5.15(2)	5.00(1)
10	5.85(3)	5.80(2)	5.70(1)
11	5.25(1)	5.20(2)	5.10(1)
12	5.65(3)	5.55(2)	5.45(1)
13	5.60(3)	5.35(1)	5.45(2)
14	5.05(1)	5.00(2)	4.95(1)
15	5.50(2.5)	5.50(2.5)	5.40(1)
16	5.45(1)	5.55(3)	5.50(2)
17	5.55(2.5)	5.55(2.5)	5.35(1)
18	5.45(1)	5.50(2)	5.55(3)
19	5.50(3)	5.45(2)	5.25(1)
20	5.65(3)	5.60(2)	5.40(1)
21	5.70(3)	5.65(2)	5.55(1)
22	6.30(2.5)	6.30(2.5)	6.25(1)
$R_1 = 53$	$R_2 = 47$	$R_3 = 32$	

Source: W. F. Woodward (1976).

Note that observations can be converted to ranks for testing more complicated models

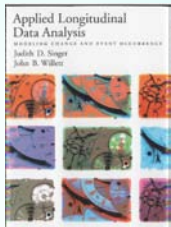
Slide 46 Rank within subject

NOTES:

Multi-level longitudinal models

www.ats.ucla.edu/stat/examples/alda

- Multilevel longitudinal models are an increasingly powerful method for analyzing longitudinal data
- Fits model parameters using maximum likelihood estimators, allows fitting of more complex variance/covariance matrices



Slide 47 Multi-level longitudinal models

NOTES:

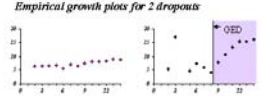
Is the individual growth trajectory discontinuous?

Wage trajectories of male HS dropouts

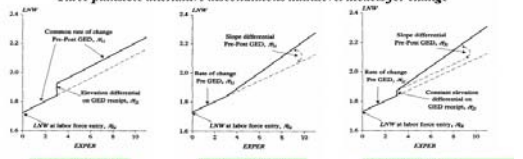
Murnane, Roemer, & Willett (1999):

- Used NLSY data to track the wages of 888 HS dropouts
- Number and spacing of waves varies tremendously across people
- 40% earned a GED
- R^2 : Does earning a GED affect the wage trajectory, and if so how?

Empirical growth plots for 2 dropouts



Three plausible alternative discontinuous multilevel models for change



$$Y_{it} = \pi_{0i} + \pi_{1i}EXPER_{it} + \pi_{2i}GED_{it} + \epsilon_{it}$$

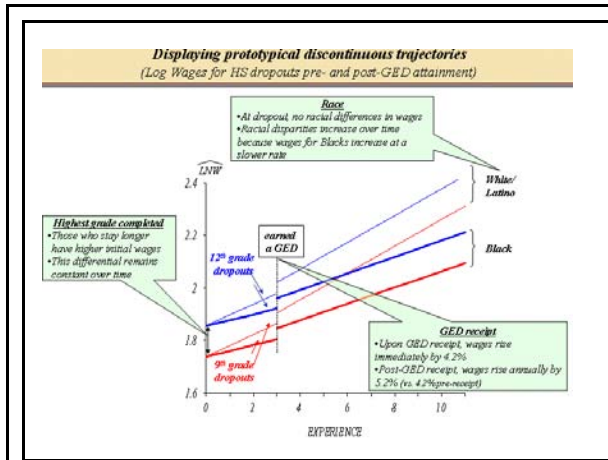
$$Y_{it} = \pi_{0i} + \pi_{1i}EXPER_{it} + \pi_{3i}POSTGED_{it} + \epsilon_{it}$$

$$Y_{it} = \pi_{0i} + \pi_{1i}EXPER_{it} + \pi_{1i}GED_{it} + \pi_{2i}POSTGED_{it} + \epsilon_{it}$$

Level-2: π 's = f(Highest Grade Completed, Ethnicity)

Slide 48

NOTES:



Slide 49

NOTES:

- ### Four important advantages of modern longitudinal methods
- You have much more flexibility in research design**
 - Not everyone needs the same rigid data collection schedule—cadence can be person specific
 - Not everyone needs the same number of waves—can use all cases, even those with just one wave!
 - You can identify temporal patterns in the data**
 - Does the outcome increase, decrease, or remain stable over time?
 - Is the general pattern linear or non-linear?
 - Are there abrupt shifts at substantively interesting moments?
 - You can include time varying predictors** (those whose values vary over time)
 - Participation in an intervention
 - Family composition, employment
 - Stress, self-esteem
 - You can include interactions with time** (to test whether a predictor's effect varies over time)
 - Some effects dissipate—they wear off
 - Some effects increase—they become more important
 - Some effects are especially pronounced at particular times.

Slide 50

NOTES:

Maximum likelihood estimation

Introduced by Fisher

- Fit iteratively with an initial estimate and adjustment of parameter estimates
- Stop the iterative fit when there is no increase in likelihood
- Could be fit with brute force, but scoring method or Newton-Raphson used
- Not based on least squares; but least squares produces ML estimators if assumptions met

Probability model for a single binary response:

$$Pr\{Y=y\} = \pi^y (1-\pi)^{1-y}$$

Probability model for n independent binary responses:

$$Pr\{Y=y\} = \pi_1^{y_1} \dots \pi_n^{y_n} \prod_{i=1}^n (1-\pi_i)^{1-y_i}$$

Slide 51 Maximum likelihood estimation

NOTES:

Slide 52 Display 21.12, page 631

Display 21.12, page 631

12 red & black marbles in a bucket, 5 marbles drawn & 3 red marbles found: What is the maximum likelihood estimate for the number of red marbles in the bucket?

Probability distributions and likelihood functions for the bucket with twelve marbles and five draws

Number (j) of Red Marbles Found in the Sample	Number (k) of Red Marbles in the Bucket			
	0	1	2	3
0	0	0	0	0
1	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000
6	0.0000	0.0000	0.0000	0.0000
7	0.0000	0.0000	0.0000	0.0000
8	0.0000	0.0000	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000
11	0.0000	0.0000	0.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000

Hypergeometric distribution:
 Gallagher's matlab m.file
 $k=[0:5];$
 $pr=LMTheorem030301(5,k,7,12)$
 $pr = 0.0013 \ 0.0442 \ 0.2652$
 $0.4419 \ 0.2210 \ 0.0265$

Display 21.13

The maximum likelihood estimator for the unknown number of red marbles in a bucket of twelve, based on a sample of five

Observed number of reds in sample (k):	0	1	2	3	4	5
$\hat{\theta}$ = ML estimate of number of reds in bucket:	0	2	5	7	10	12

NOTES: