Numbers and Functions

Math 140 - Calculus I

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UMass Boston

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Numbers

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- Numerical Functions

$$f\colon \mathbb{R} \to \mathbb{R} \quad \text{OR} \quad f\colon A \to \mathbb{R} \text{ with } A \subset \mathbb{R}$$

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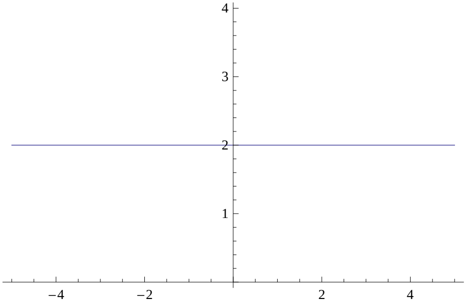

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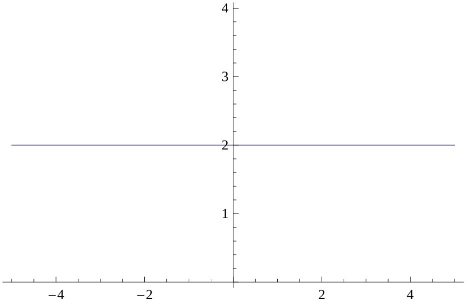

- Important, but boring ...

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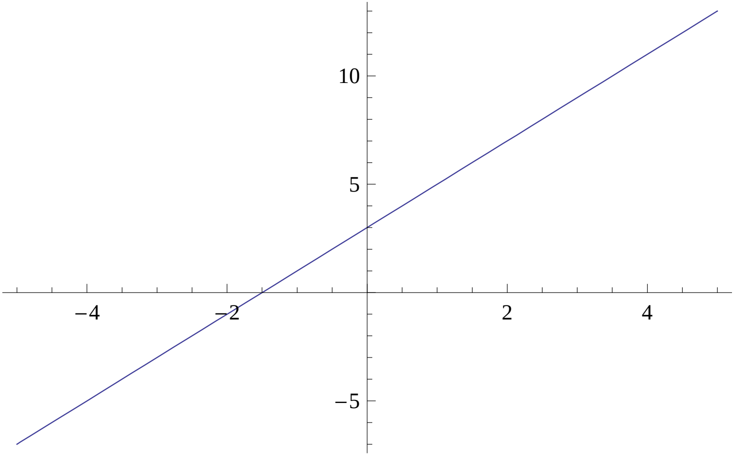
10
5
−5
−4
−2
2
4

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  - Non-example: $f(x) = 4x^3 + x^{-2}$

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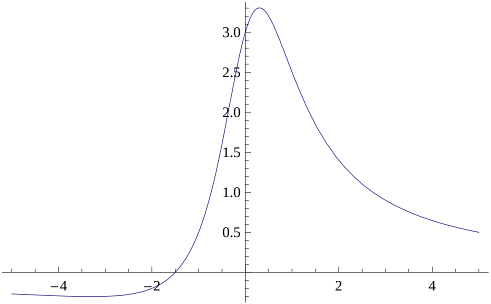

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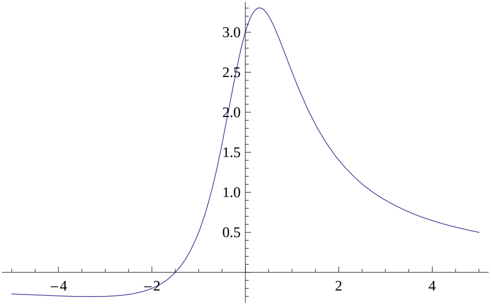

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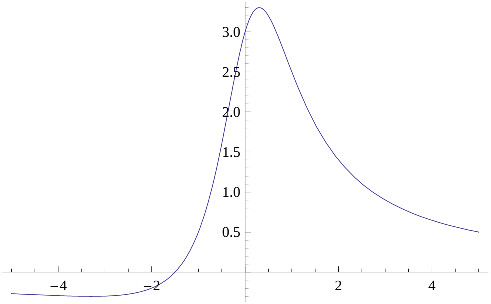

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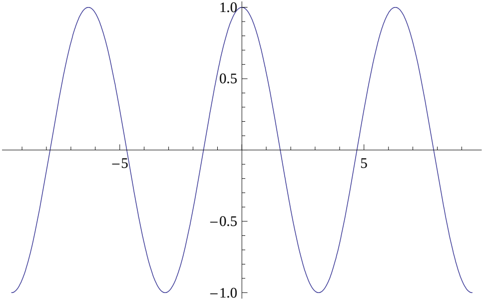
- 

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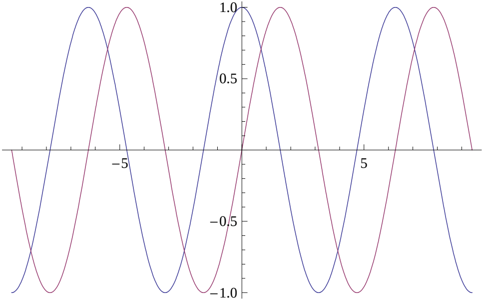
- 

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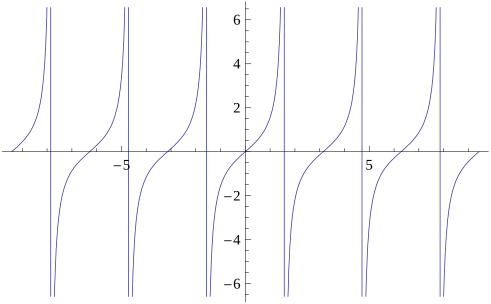
- 

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- Periodicity $f(x+T) \equiv f(x)$ for all x.

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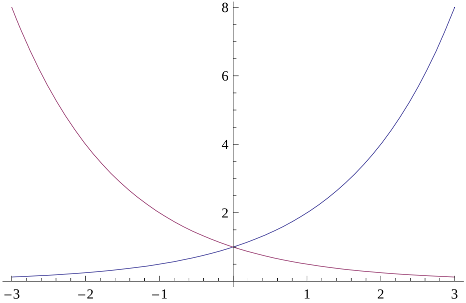

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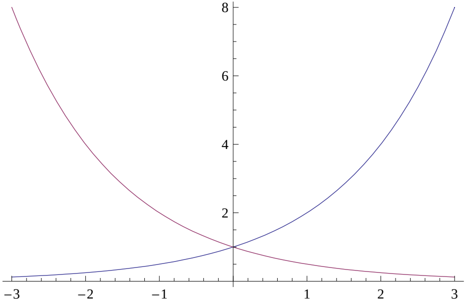

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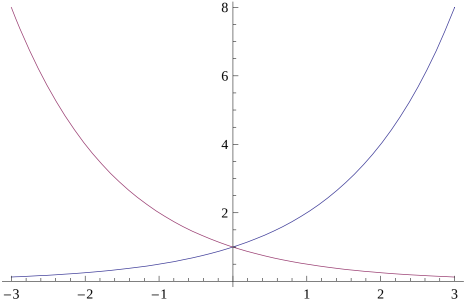

- Long term behavior
- What does π^x really mean?

Logarithmic functions

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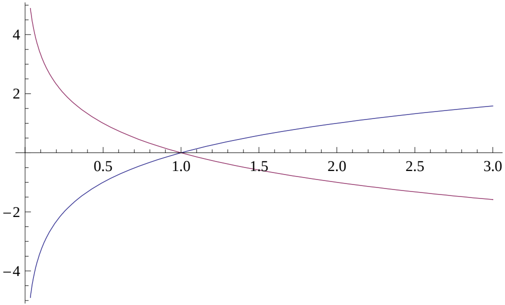

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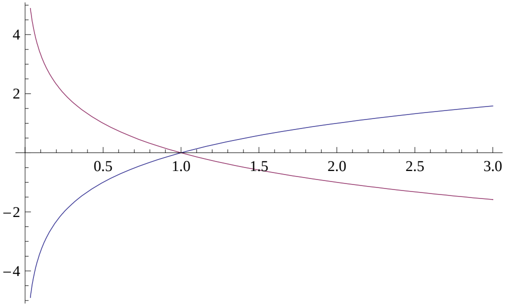

- Exponentials and logarithms

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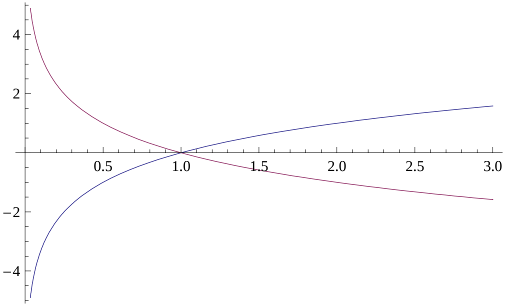

- Exponentials and logarithms

$$a^{\log_a x} = x \qquad \log_a(a^x) = x$$