UMass Boston, Fall 2009 — Tuesday, September 29, 2009
Math 140: Calculus I — Lecture # 11
Notes by: Catalin Zara — **Topic: Practice Exam**

(1) **(5 points)** True or false? (**Do not guess**: for every correct answer you get one point, but for every incorrect answer you lose one point. If you are not sure about the correct answer, do not answer the question).

(a) ( ) If x is a nonzero number, then $x^{1/3} = \frac{1}{x^3}$.

(b) ( ) If f is any function, then $f(x+y) = f(x) + f(y)$.

(c) ( ) If $f(x) = 2^{x-1}$ and $g(x) = 2x$, then $(f \circ g)(x) = 2^{2x-1}$.

(d) ( ) If f is any function such that $\lim_{x\to 1} f(x) = 3$, then $f(1) = 3$.

(e) ( ) If f is any function such that $\lim_{x\to 1} f(x) = 3$, then $\lim_{x\to 1^+} f(x) = 3$.

(2) **(24 points)** Find the following limits. You will not receive any credit if you do not justify your answers.

$$\lim_{x\to 4} \frac{x^2-4x}{x^2-3x-4} \qquad \lim_{x\to 3^-} \frac{x+3}{x^2-9} \qquad \lim_{x\to 0} \frac{\sqrt{3x+1}-1}{5x} \qquad \lim_{x\to\infty} \frac{2x+1}{3x-x^2}$$

(3) **(18 points)** A function f is defined by

$$f(x) = \begin{cases} \frac{x^2-6x+8}{x-4}, & \text{if } x < 4, \\ b, & \text{if } x = 4, \\ ax-2, & \text{if } x > 4. \end{cases}$$

For what values of the constants a and b is the function f

(a) continuous at $x = 4$? Justify your answer.

(b) differentiable at $x = 4$? Justify your answer.

(4) **(15 points)** Show that the equation $x^4 - x - 1 = 0$ has at least one real solution. Find an interval of length 0.5 that contains a solution of the equation. Justify your answer.

(5) **(18 points)** Sketch the graph of a function $f\colon \mathbb{R} \to \mathbb{R}$ that satisfies all of the following conditions:

- f has a horizontal asymptote at $-\infty$ and a vertical asymptote at $x = 2$;
- $\lim_{x\to\infty} = -\infty$ and $\lim_{x\to 4^-} f(x) = 2$
- f has a removable discontinuity at $x = 3$ and a jump discontinuity at $x = 4$
- f is continuous at 0 and $\lim_{x\to 0} f(x) = 3$.

(6) **(18 points)** Let $f(x) = \sqrt{3x+1}$.

(a) **Use the limit definition of the derivative** to find $f'(1)$.

(b) Find the equation of the tangent line to the graph of f at the point corresponding to $x = 1$.